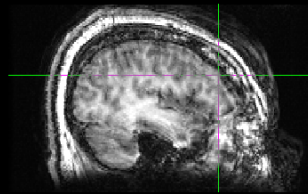
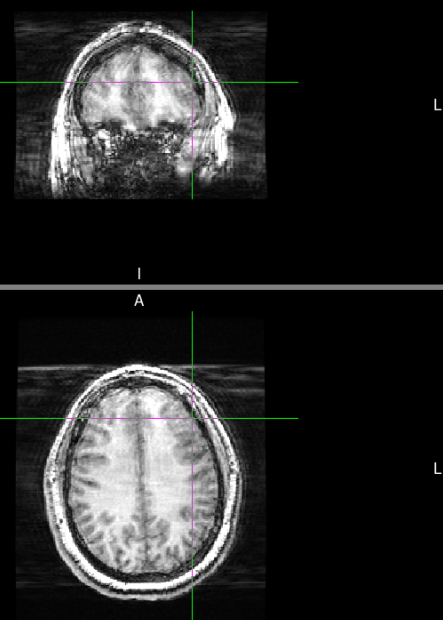


# Preprocessing and artefacts



Instructor:  
Luis Hernandez-Garcia, Ph.D.  
Research Professor  
FMRI Laboratory, Biomedical Engineering

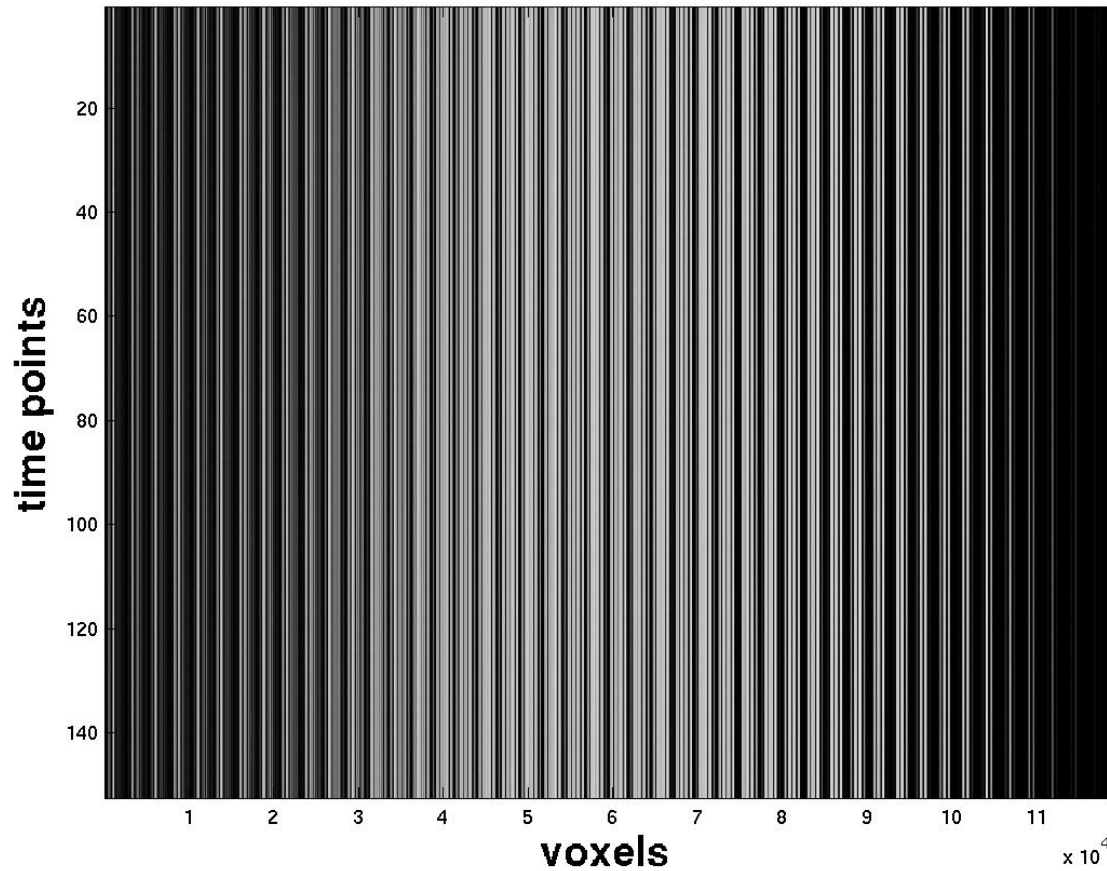
# FMRI analysis in broad strokes

1. Formulate a 4D model of activation.
2. Examine the time-series at **every voxel** : Fit your model to the data.
  1. Does the model fit?
  2. Make some statistical images
3. Compare statistical images between subjects, groups ....

# Typical data arrangement

- One 3D image per time point
- Most popular: NIFTI format (.nii)
  - header information
  - the image itself
  - (It's almost the same as AVW or Analyze format)
- Useful to read all the data as a matrix
  - Rows = time
  - Columns = space (collapsed to 1 dimension)
  - (... or vice versa)

# Example





# Implicit Assumptions in Analysis

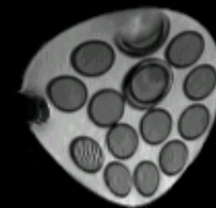
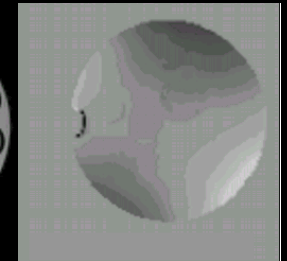
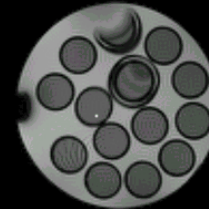
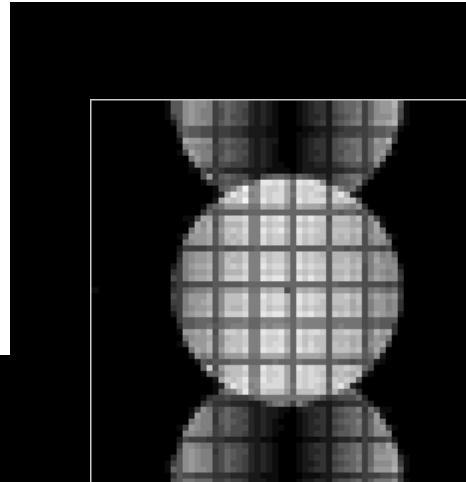
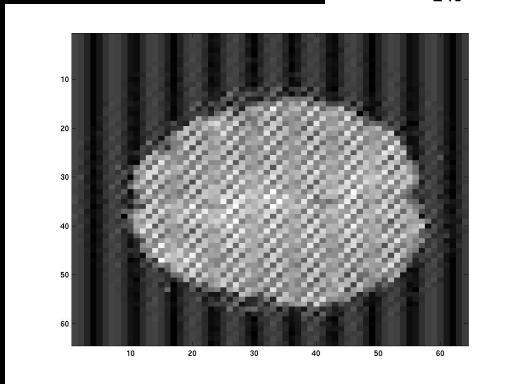
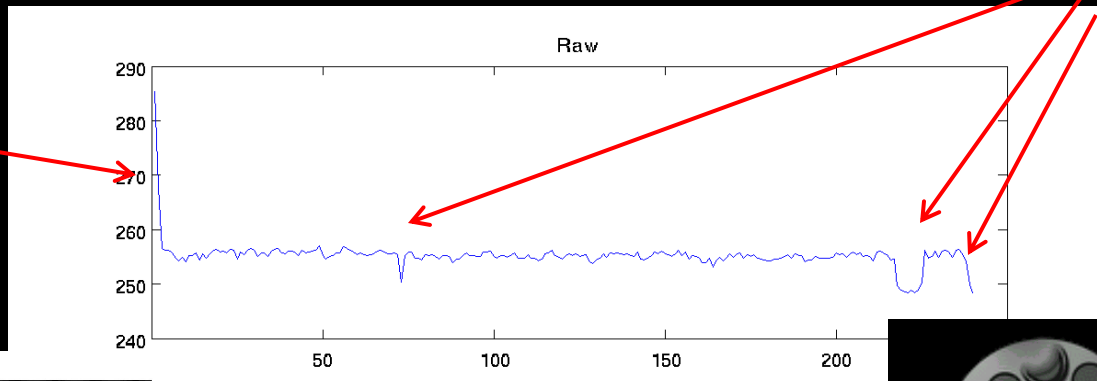
- Each Voxel contains a time series from that voxel ONLY
- All voxels in a given 3D image are sampled at the same time
- All brains are morphologically identical
- Paradigm is the SOLE SOURCE of variance in the time series.
- The image corresponds Exactly to the anatomy

**Wishful Thinking!**

# The harsh reality

T1 settling time

Scanner spiking issues



# Lecture Goals

- Understand the following confounds in fMRI and what corrections exist
  - Slice Timing effects (temporal shifting)
  - Movement (rigid body realignment)
  - Physiological Artifacts: respiration and heart beat (regression filters)
  - Electronic Noise (filters and autoregressive models)
  - Morphology (non-linear warping)
  - Image distortions (susceptibility, ghosting, off-resonance)

# Lecture Goals (II)

As a side effect, you will be introduced to signal and image processing concepts:

1. Linear transformations
2. Basis functions
3. Sampling, re-sampling, interpolation
4. Optimization, cost functions
5. other side effects include drowsiness, nausea

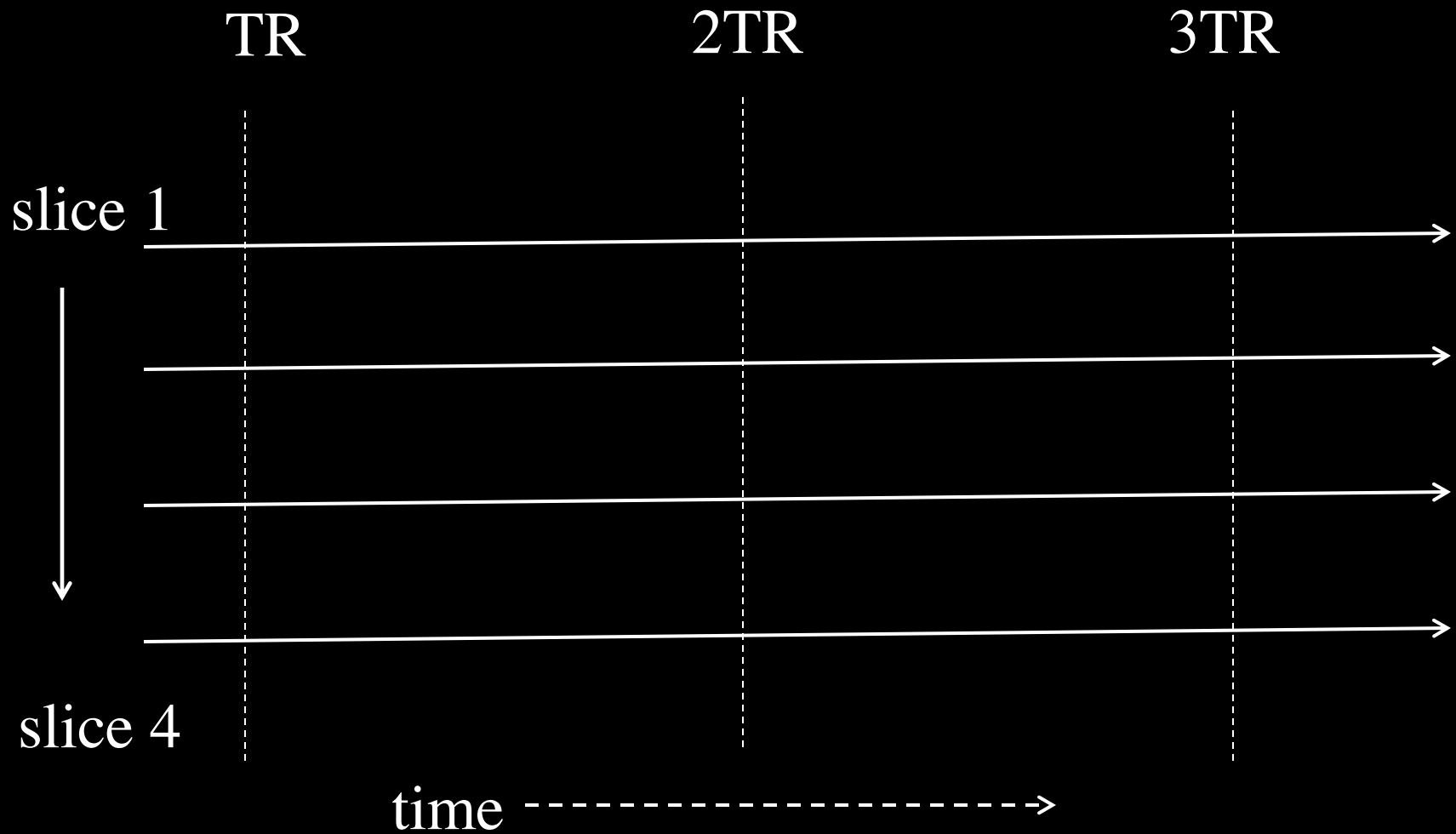
# Lecture Goals

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  - Morphology (non-linear warping)

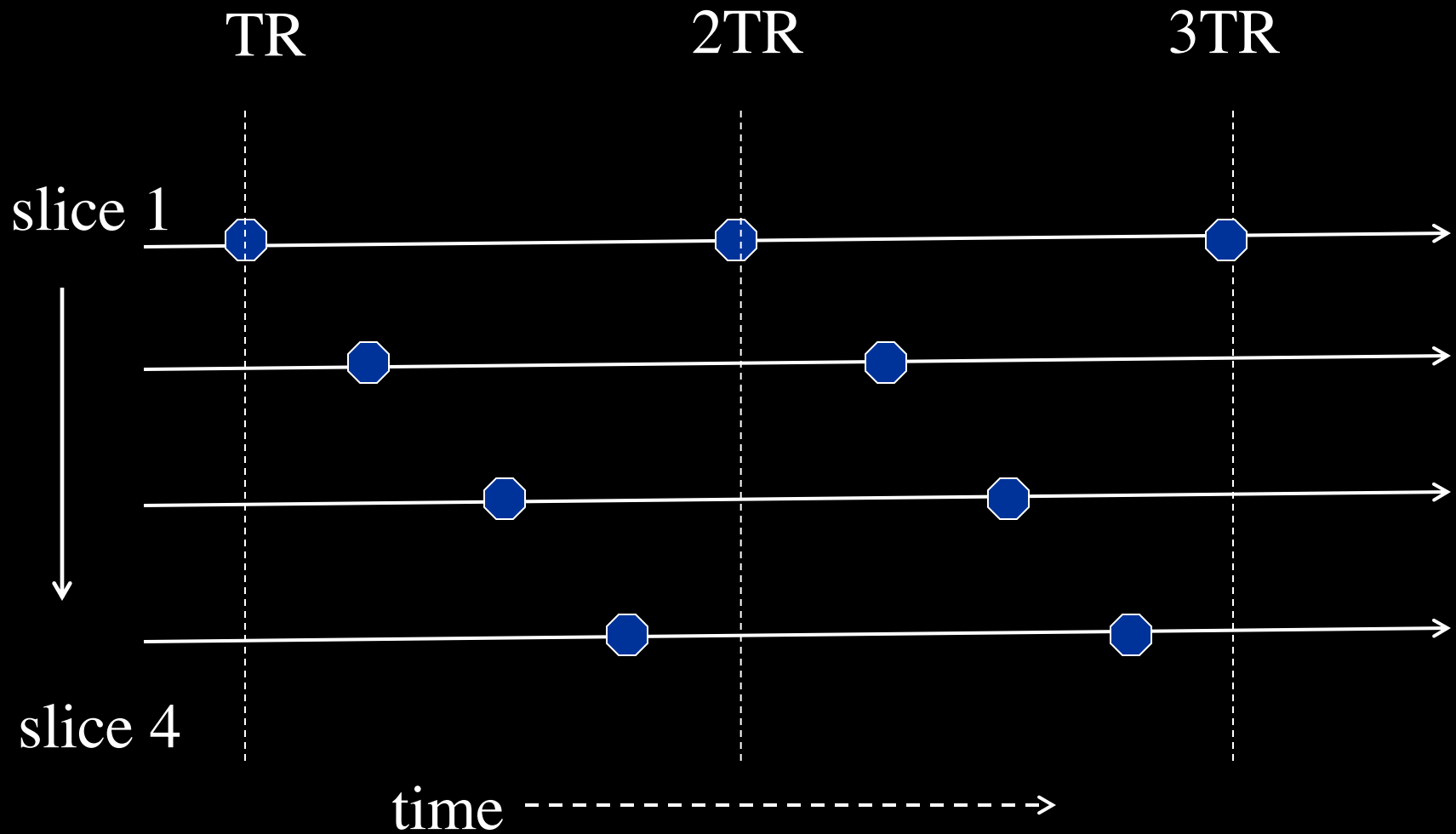
# Timing Errors

- MR images are typically collected one slice at a time (exceptions: 3D imaging, multi-band imaging)
- The slices can be collected sequentially or interleaved. This is also true in Multi-band imaging.
- Delay between slice excitations is typically
$$= TR / (num. slices)$$
- Therefore, the time series are time-shifted differently in each slice

# FMRI data “layout”

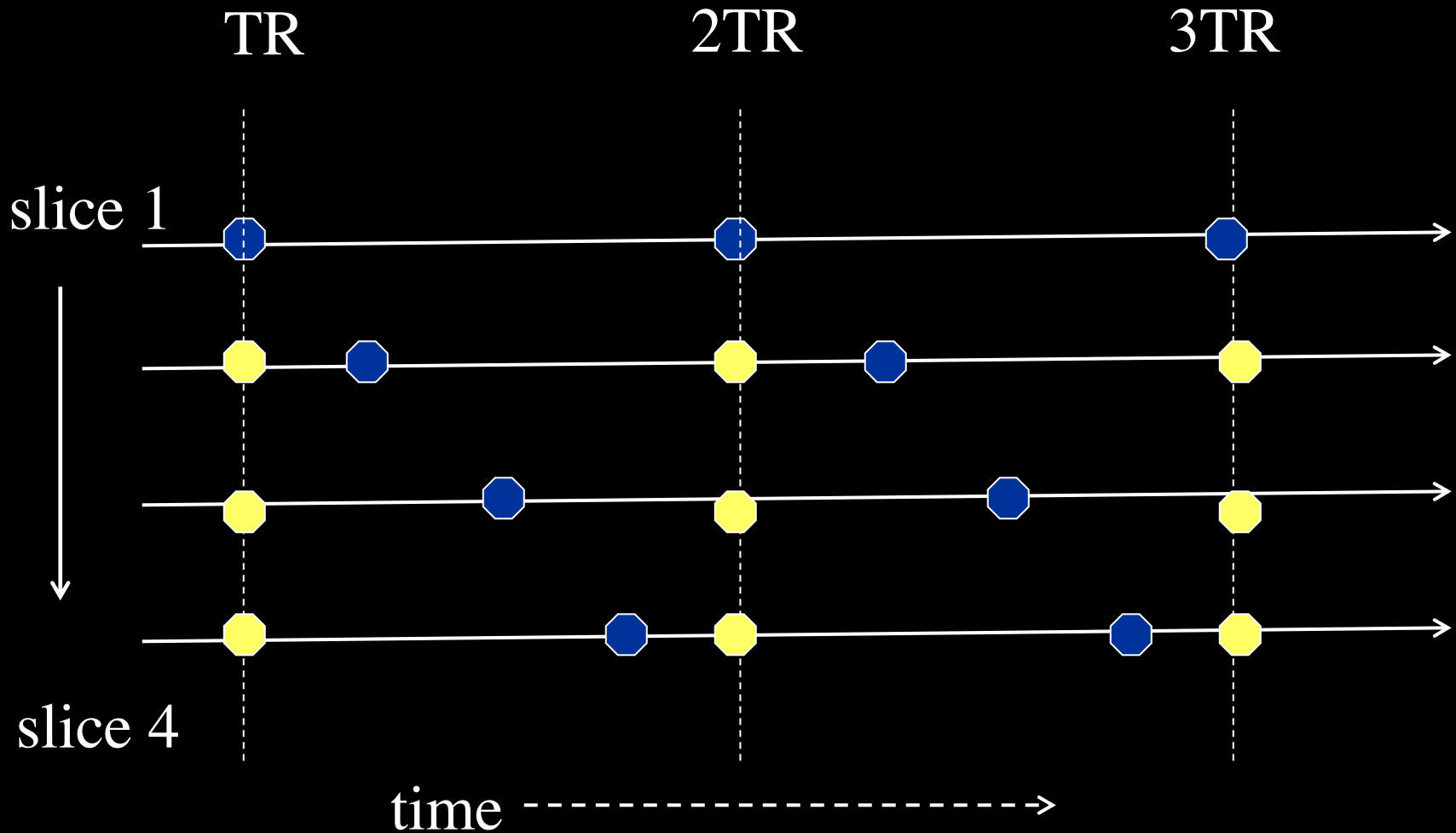


# Acquisition





# Acquisition



# Sampling Error in Time

The data you think you have

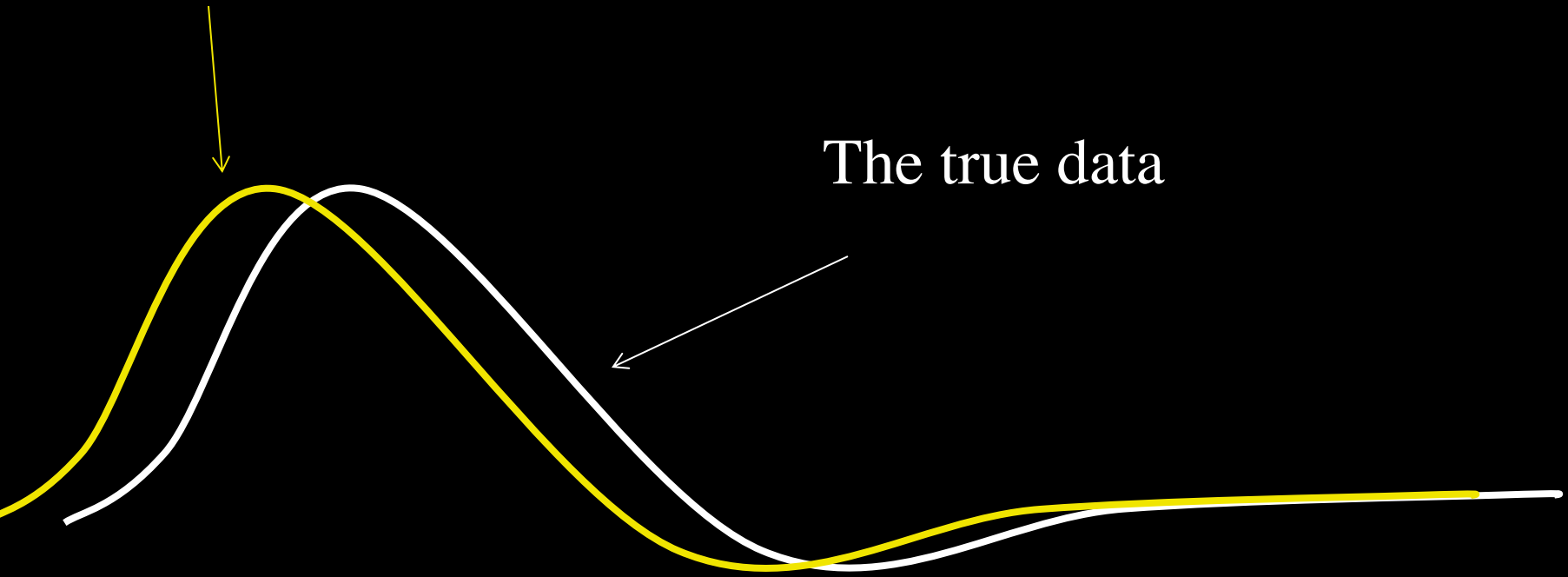
The data you really have



# Sampling Error in Time

How the data looks

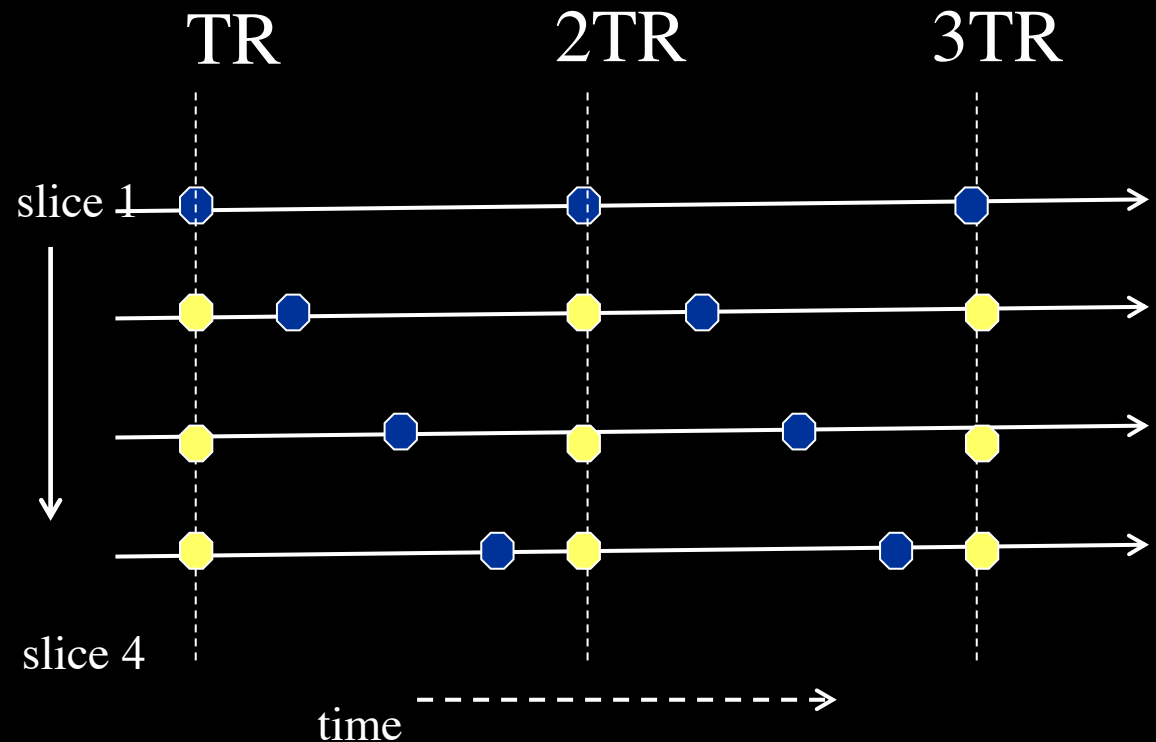
The true data



so shift it back!

# Interpolation / Temporal shifting

- Time shift is the same as interpolation
- Interpolation: calculate a missing data point from its neighbors
- *Interpolation = weighted average*



# Interpolation = Temporal shift by a fraction of a sample

*Time domain*

*Frequency Domain*

- Have:

$$f(t)$$

$$F(\omega)$$

- Want:

$$f(t - \tau)$$

$$e^{-i\omega\tau} F(\omega)$$

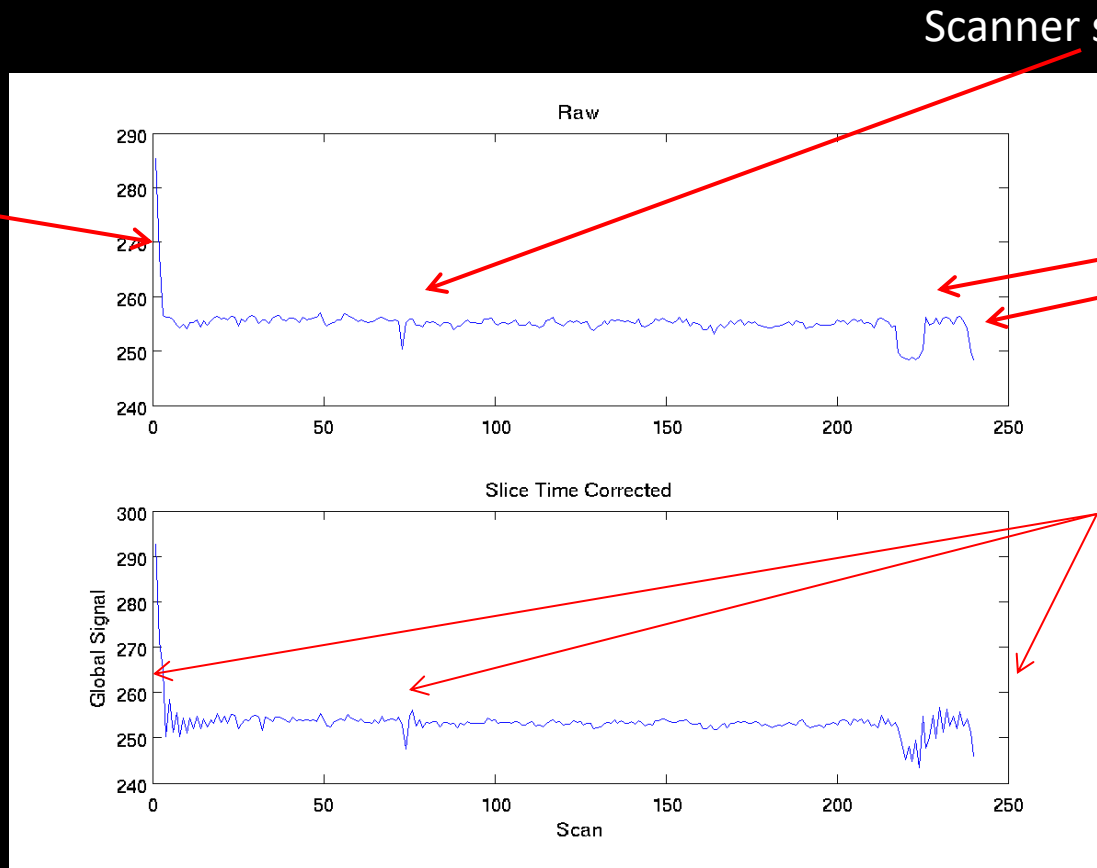
# Temporal shifting strategy

1. Fourier transform along time dimension
2. Add linear phase increment (multiply the complex data by  $e^{-i\phi\omega}$  )
3. Inverse Fourier transform
4. Note: this strategy uses the whole time series. Noise in one sample can contaminate the whole time course!

# Interpolation side effects

T1 settling time

Global signal (mean over space): a good indicator of large scale changes in signal

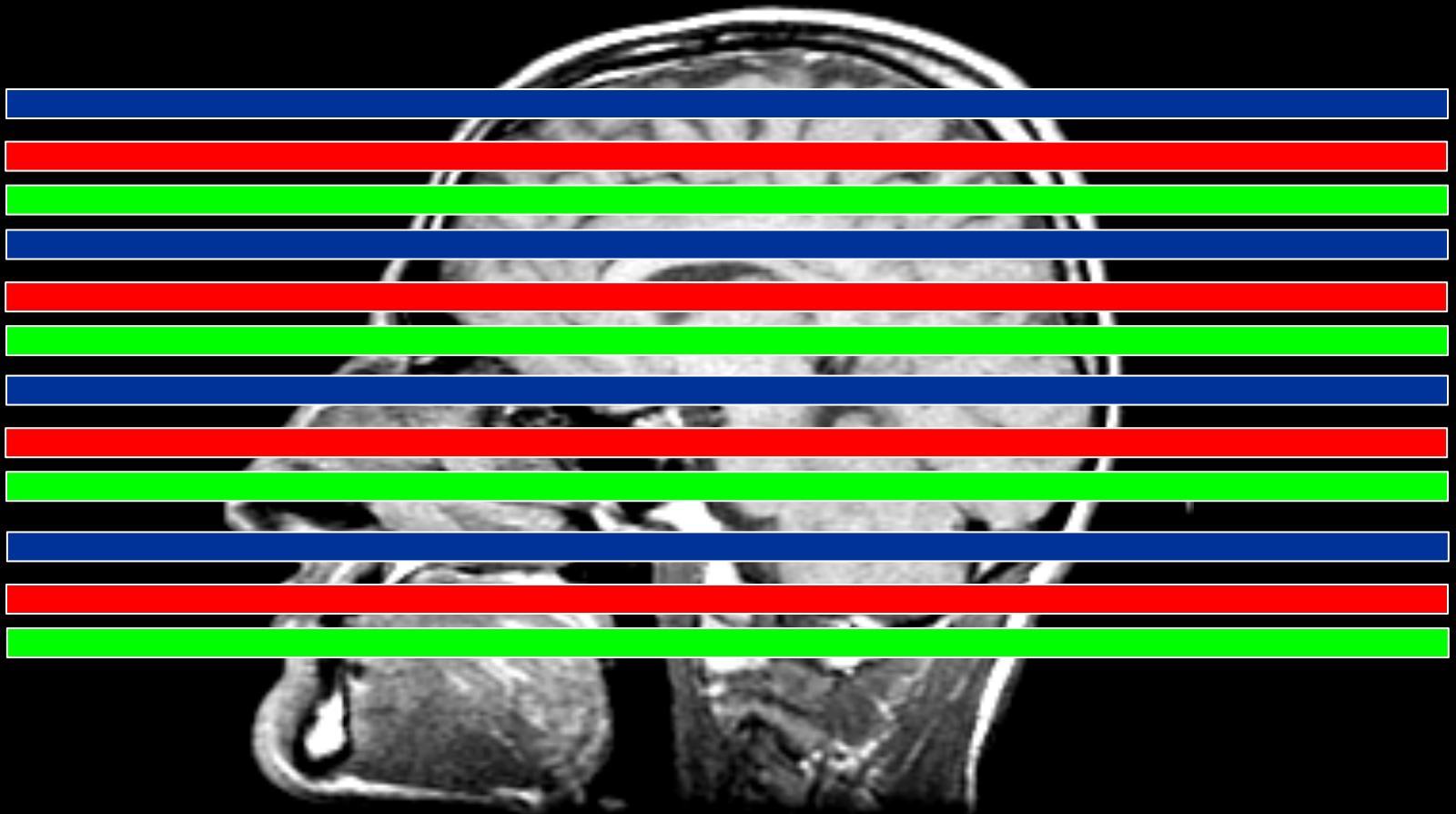


Scanner spiking issues

Subject movement

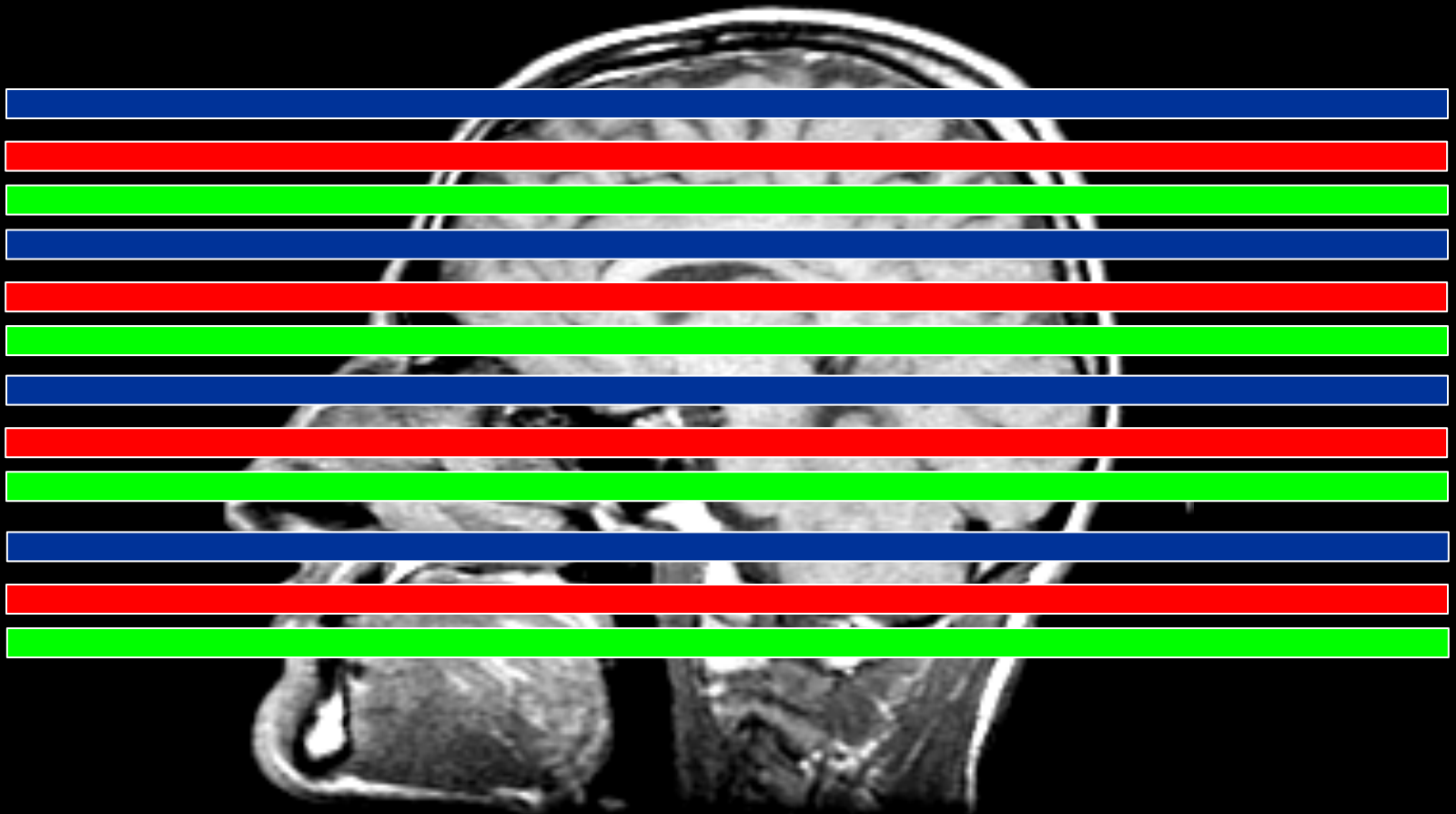
Interpolation from Slice timing correction "spreads" artifacts over time!

# What about Multi-Band imaging?





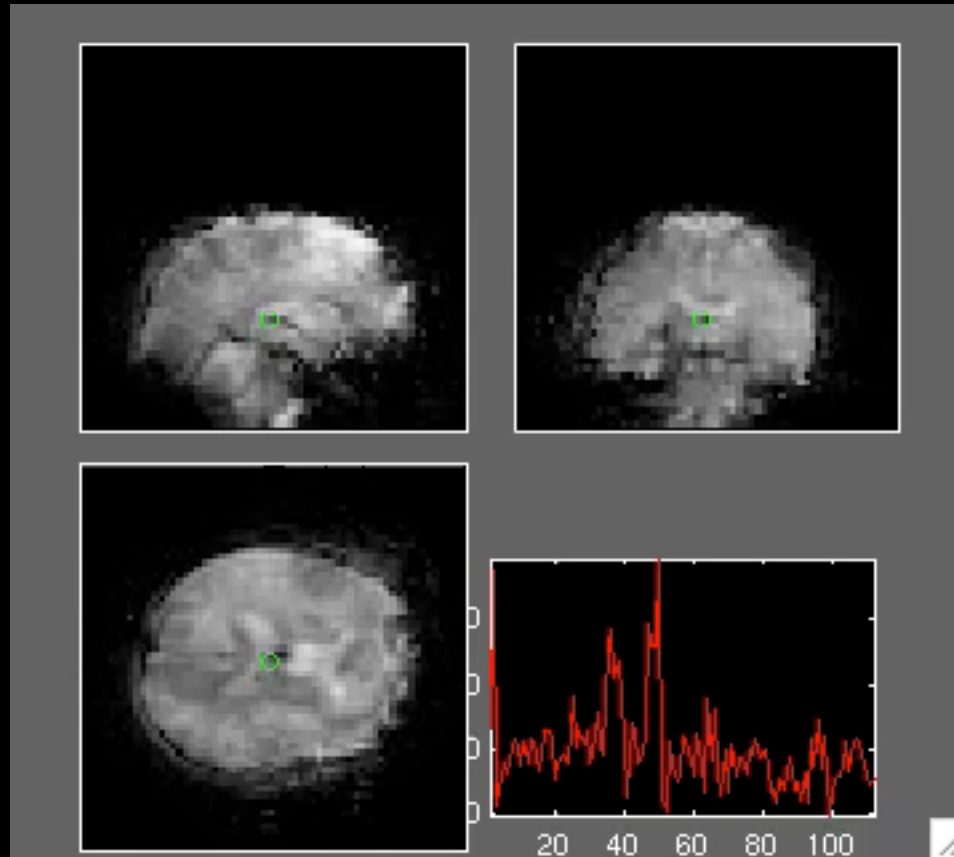
# What about Multi-Band imaging?



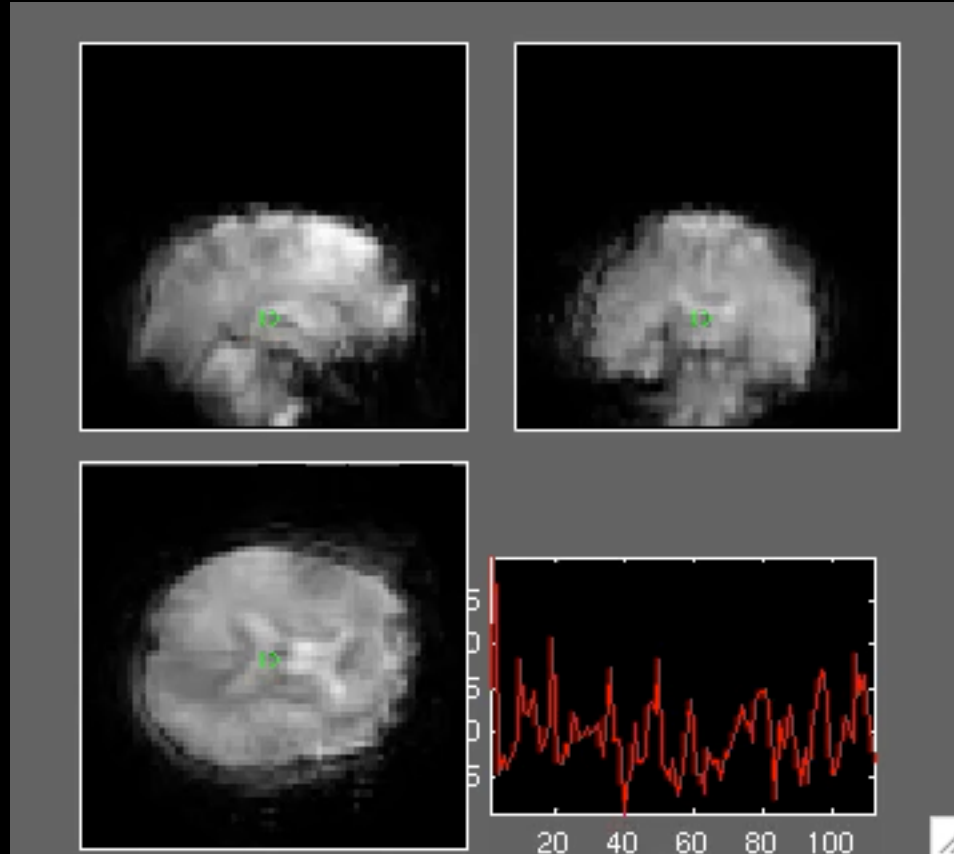
# Lecture Goals

- Understand the following confounds in fMRI and what corrections exist
  - Slice Timing effects (temporal shifting)
  - Movement (rigid body realignment)
  - Physiological Artifacts: respiration and heart beat (regression filters)
  - Image distortions (susceptibility, ghosting)
  - Electronic Noise (filters and autoregressive models)
  - Morphology (non-linear warping)

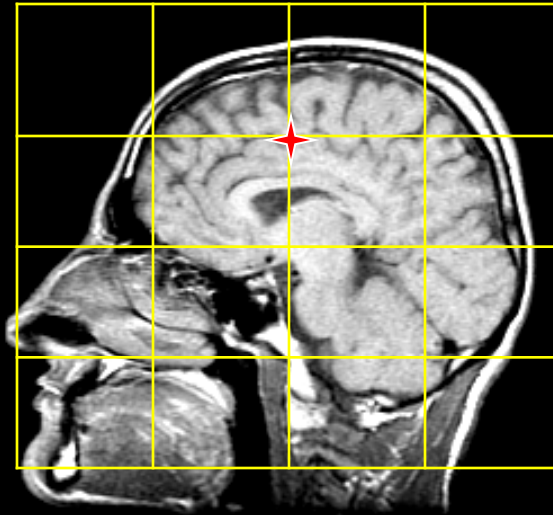
# Movie: Uncorrected movement



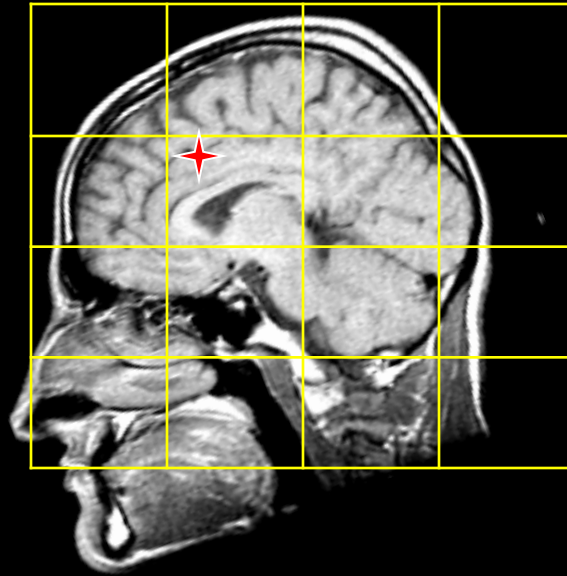
# Movie: Corrected Movement



# Movement

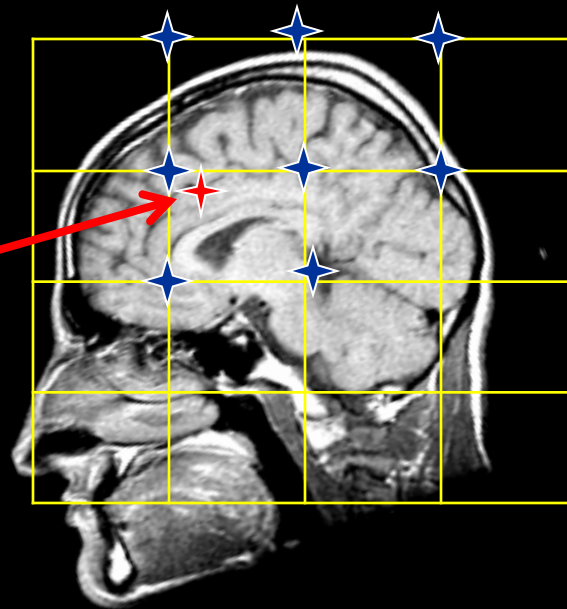


# Movement



# Movement

Interpolate this  
point from its  
neighbors



# Resampling the image

- Think of realignment as transforming the sampling grid, rather than the image.
- Interpolation:
  - Choose weighting function (kernel):
    - Nearest neighbor
    - bi-linear, tri-linear interpolation
    - sinc interpolation



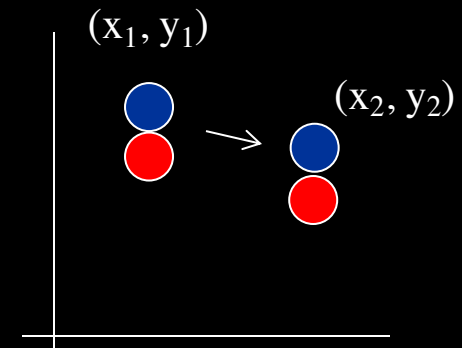
# Movement: figuring out the new coordinates

In 2 Dimensions:

- shift from  $(x_1, y_1)$  to  $(x_2, y_2)$ :

$$x_2 = x_1 + \Delta x$$

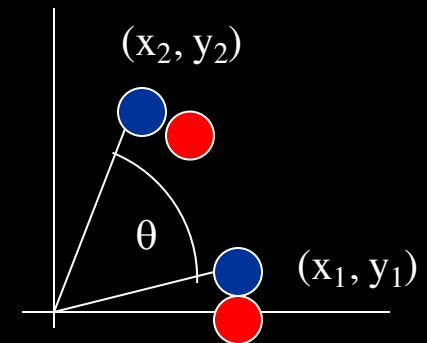
$$y_2 = y_1 + \Delta y$$



- Rotation from  $(x_1, y_1)$  to  $(x_2, y_2)$ :

$$x_2 = x_1 \cos(\theta) + y_1 \sin(\theta)$$

$$y_2 = -x_1 \sin(\theta) + y_1 \cos(\theta)$$



# 2-D Transformation matrix

- Both Together (note that the order matters)

$$x_2 = x_1 \cos(\theta) + y_1 \sin(\theta) + \Delta x$$

$$y_2 = -x_1 \sin(\theta) + y_1 \cos(\theta) + \Delta y$$

...but let's say it with matrices:

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & \Delta x \\ -\sin(\theta) & \cos(\theta) & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

# 2-D Transformation matrix

$$(x_2, y_2) = A(x_1, y_1)$$

this extends to N-dimensions too

# 3-D Rotation matrices

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ \cos(\theta) & 1 & \sin(\theta) & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

xy plane  
rotation

xz plane  
rotation

yz plane  
rotation

# Estimation of Movement

1. Compare the two images. How good is the match?
2. Choose a set of translations, rotations
3. Combine the six transformations matrices (linear operators) into one “rigid body” transformation

$$r_2 = A r_1$$

3. Resample the images at the new locations
4. Are the two images more alike (do they line up better)?
5. Repeat and search for the best matrix  $A$

# Comparing images: cost function

- How do you know two images match?
  1. Least squares difference

$$\sum (I_1 - I_2)^2$$

2. Normalized correlation, correlation ratio

$$\frac{\sum(I_1 I_2)}{(\text{Var}(I_1) (\text{Var}(I_2))^{1/2})} \quad \frac{\text{Var}(E[I_1 I_2])}{\text{Var}(I_2)}$$

3. Mutual information

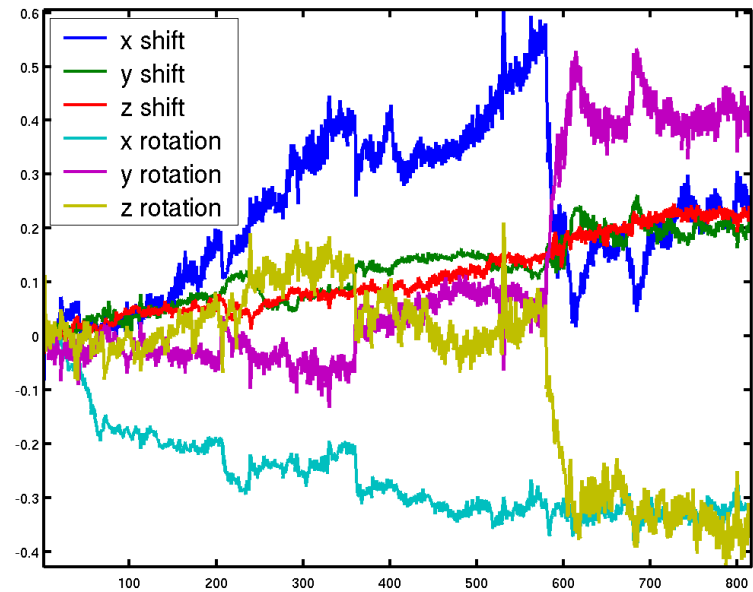
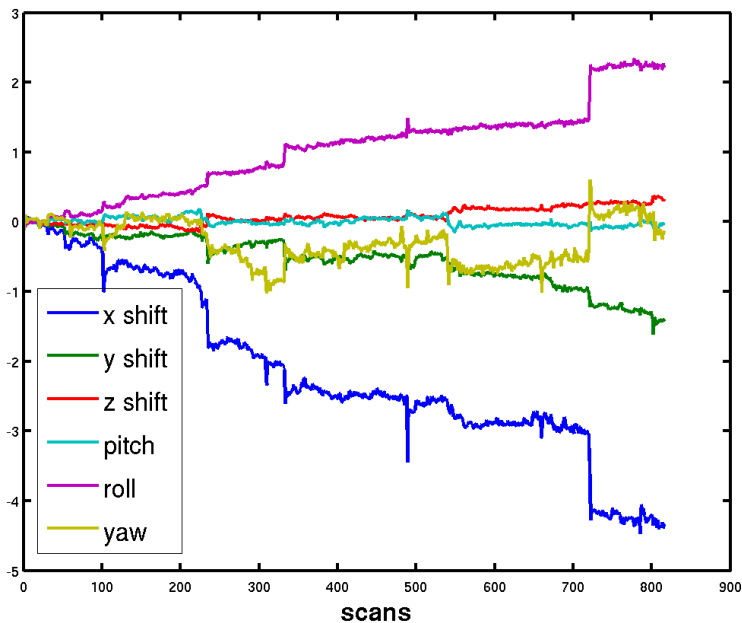
$$M(I_1, I_2) = \sum_{i,j} p(I_1, I_2) \log_2 \left( \frac{p(I_1, I_2)}{p(I_1)p(I_2)} \right)$$

4. others ...M. Jenkinson and S.M. Smith. Medical Image Analysis, 5(2):143-156, June 2001

# Search Strategies

- Least squares ( $Y=X\beta$ ) ... ?
- Steepest descent: vary parameters and compute the gradient in the cost function (error). Keep going as long as it gets better.
- There are variations on this theme:
  - simplex
  - Newton's method / gradient descent
  - Adaptive methods
  - others...

# Sample Movement Parameters





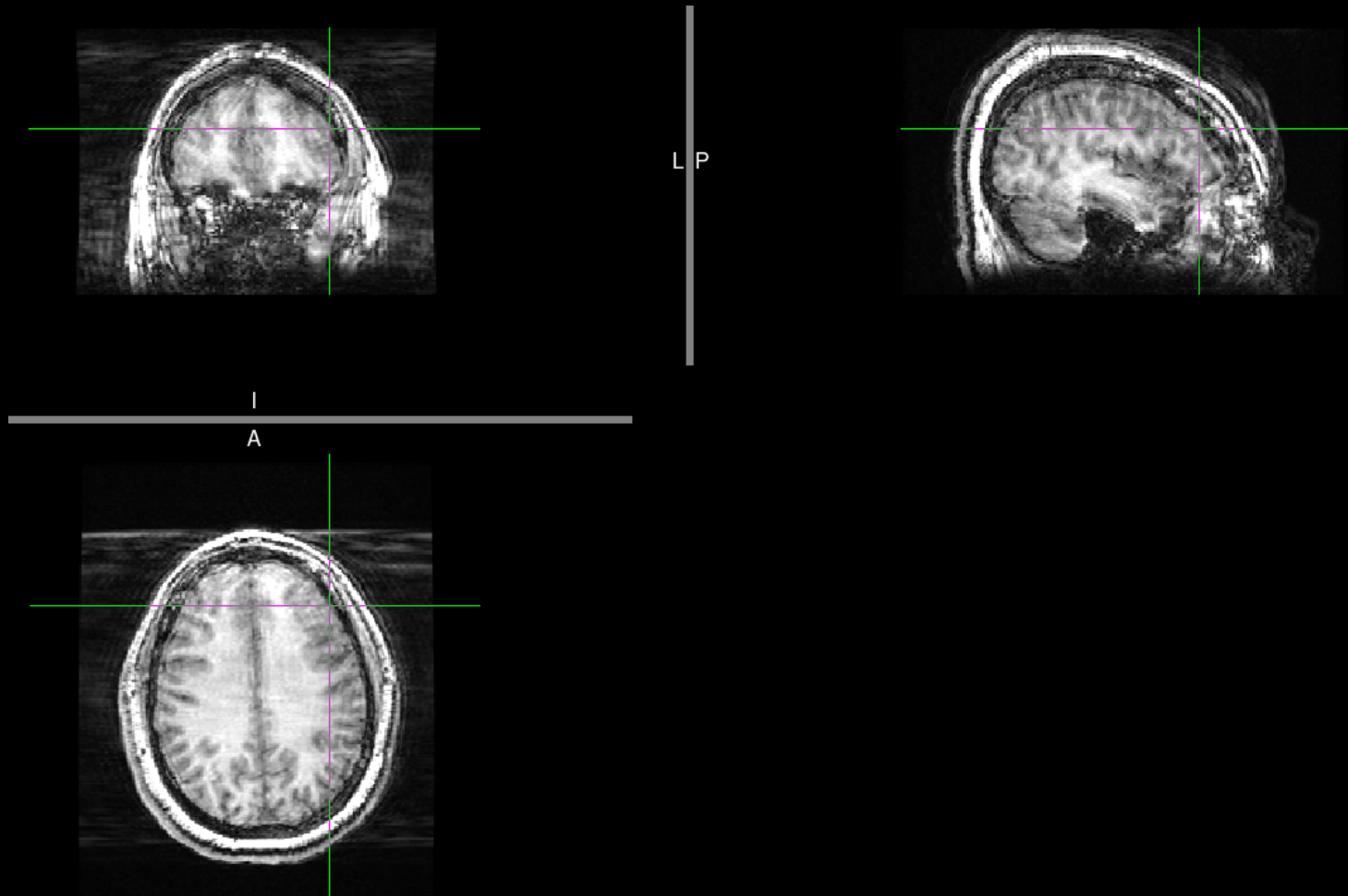
# Movement Noise

- In addition to misplacing voxels, you introduce a fluctuation in signal intensity during realignment
- This is a complicated function of the movement:
  - Movement affects the k-space trajectory
  - Mixes partial volumes,
  - Interpolation methods also have an effect on intensity.

# Movement Noise corrections

- Minimize movement while acquiring data whenever possible !!
- Including movement regressors as confounds: what is the physics relationship between signal intensity and movement?
  - It's Complicated
    - ... but can be approximated with first order linear relationship.
    - Higher order terms (Lund et al., 2005 NeuroImage) are extremely helpful
  - Reduces residual variance
  - If movement is correlated with task = **BIG TROUBLE!**

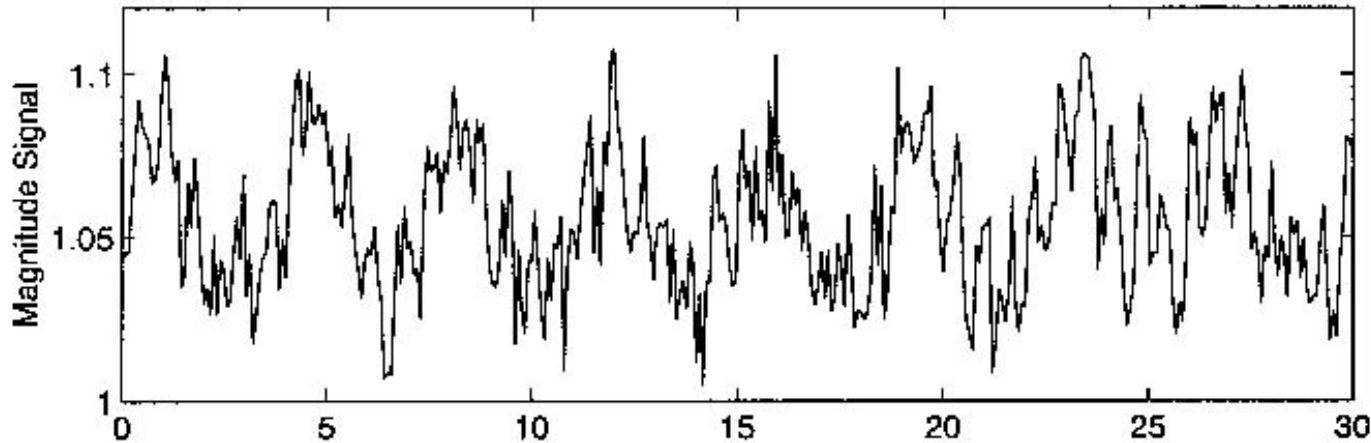
# Movement Artefacts



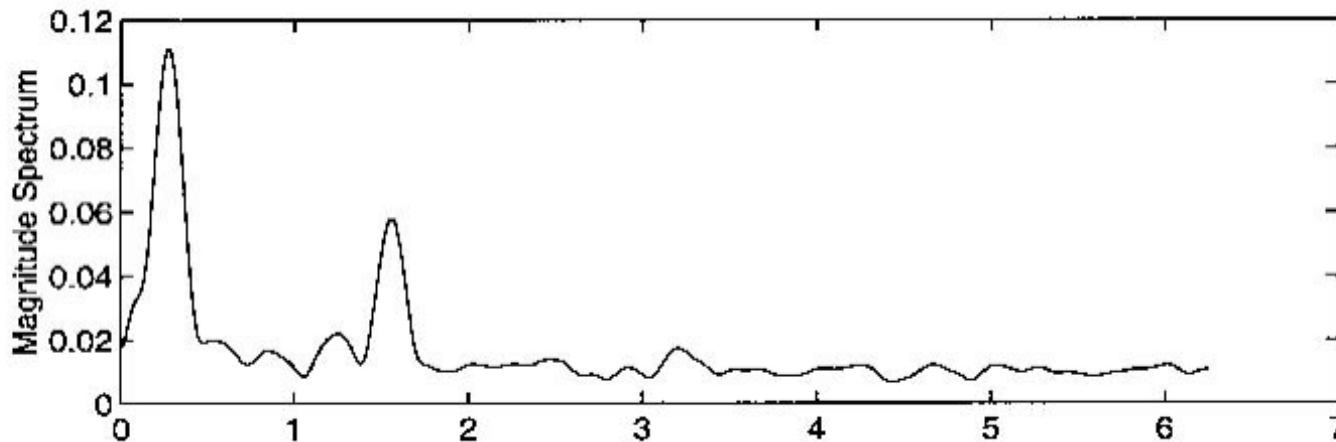
# Lecture Goals

- Understand the following confounds in fMRI and what corrections exist
  - Slice Timing effects (temporal shifting)
  - Movement (rigid body realignment)
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  - Image distortions (susceptibility, ghosting)
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  - Morphology (non-linear warping)

# Physiological oscillations



Time  
domain



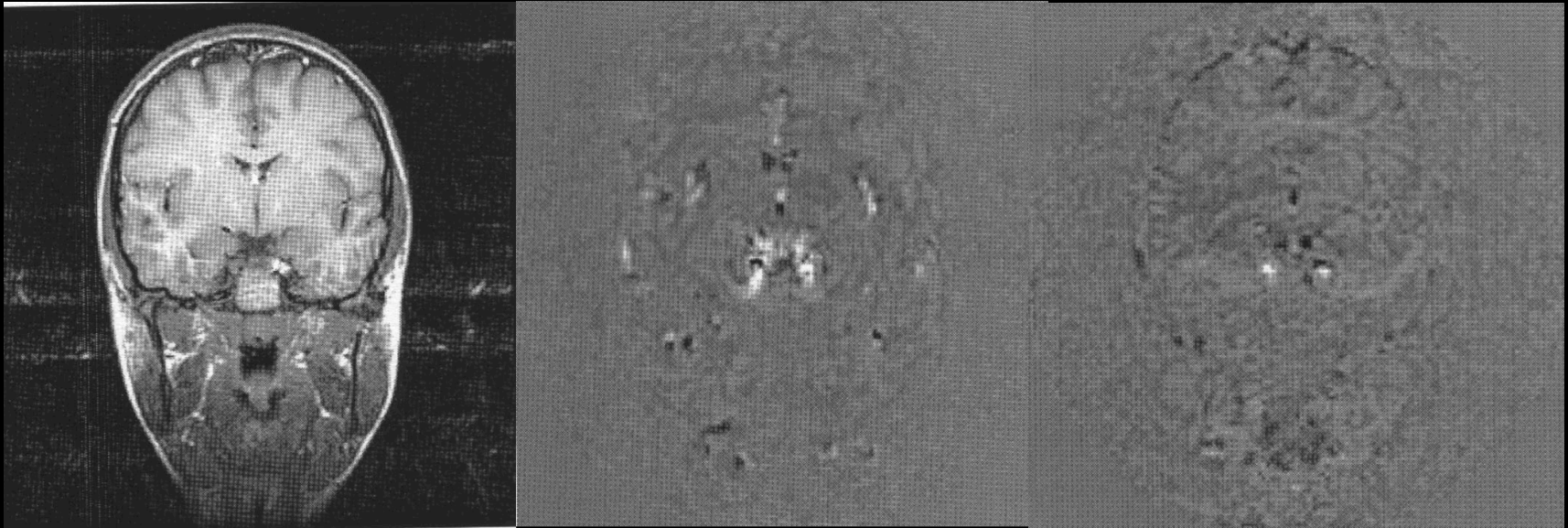
frequency  
domain

# Cardiac and Respiratory Variance

anatomy

Residual Variance w/o  
Physio correction

Residual Variance w/  
Physio correction



Data courtesy of Scott Peltier

# Cardiac Noise

- Blood flow is pulsatile -> changes blood volume, and velocity.
- How blood flow affects the MR signal:
  - Flow enhancement (incoming spins have not received any RF , fully relaxed -> more signal)
  - Flow void (sometimes spins flow so fast through the plane that they don't see the RF pulse, or they flow out before they can be encoded -> less signal )
  - Flow induced displacement (additional phase acquired because of in-plane movement -> distorted/displaced signal, ghosting)

# Reduction of cardiac effects during Acquisition

- Use a smaller flip angle - reduces flow enhancements and voids.
- Use flow “spoilers” to remove vascular signals.
  - (A pair of balanced gradient pulses, a.k.a. “crushers”, makes moving spins get out of phase with each other and cancel, but the stationary spins come back into focus and their signal remains.)
- Use fast acquisition (single shot) to reduce ghosting.
- “Cardiac Gating”



# Reduction of cardiac artifacts after acquisition

- Digital Filters ...
- Measure cardiac waveform and include in analysis as a confound.
- Note: watch out for aliasing!!
  - heartbeat  $> 1$  Hz
  - If  $TR = 1$  sec.  $\rightarrow$  Nyquist frequency  $< 0.5$  Hz
  - SMS speeds up sampling rate – reduces aliasing.

# Respiration

- Air and Tissue difference in magnetic susceptibility ( $\chi$ ) :  
Distortion of  $B_0$  field
- Chest movement changes the shape of the  $B_0$  field.
- Resonant frequency changes slightly
  - ( Recall that  $\omega_0 = \gamma B_0$ )
- Blood Pressure and CBF change slightly with respiration
  - pulsation of arteries -> CBV
  - pCO<sub>2</sub> -> CBF

# Corrections for Respiration

- Fast image acquisition (single shot)
- Record Respiratory waveform and use as a confound. (Note- sometimes it's correlated with task of interest)
- “Notch” or “band-stop” Filters
- Aliasing is not as much of a problem as in cardiac fluctuations, but might still interfere with design
  - Respiration  $\sim 0.08$  Hz
  - BOLD  $\sim$  from 0.01 to 0.05 (broad)
  - typical Nyquist frequency  $< 0.5$  Hz

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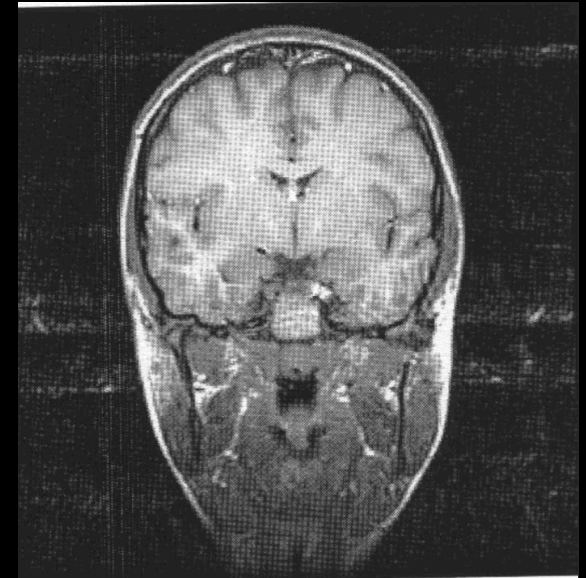
# Definitions

## **Signal to Noise Ratio( SNR) :**

Ratio of the amount of Signal to the standard deviation of the noise

## **Contrast to Noise Ratio (CNR):**

Ratio of Difference in signal between two “things” to the standard deviation of the noise



# Signal Intensity in MRI

- The signal is proportional to  $M$  and  $V$ , where:
  - $V = \Delta x \Delta y \Delta z$  is the voxel volume
  - $M$  is the intensity of the magnetization vector.
- Proton Density and  $B_0$  determine the size of the spin populations, I.e. - magnitude of  $M$
- Acquisition timing also affects the observed Signal

$$\rho(1 - e^{-TR/T1})e^{-TE/T2}$$

# Thermal Noise

- Not related to the NMR phenomenon but from random thermal fluctuations.
  - Present with or without  $B_0$ , RF, Gradients
- Uniform spectral density: “white noise”.
- Comes from the whole body – amount of noise depends on the amount of the body to which the receive coil is sensitive.

# Thermal Noise in MRI

- The noise/pixel in a 2D image is:

$$\sigma_n^2 = \frac{1}{N_x N_y} \frac{\sigma}{\Delta t} \propto \frac{1}{N_x N_y \Delta t} = \frac{1}{T_{A/D}}$$

where:

- $N_x$  and  $N_y$  are the number of samples in the  $x$ - and  $y$ -directions
- $\sigma$  is the std. dev. of the inherent noise in the system
- $\Delta t$  is the sampling time (faster sampling allows more noise into the system), and
- $T_{A/D}$  is the total time the signal is sampled ... (includes number of averages)



# Signal to Noise Ratio

- The *SNR* is then:

$$SNR \propto \frac{\text{signal}}{\sigma_n} \propto m_0 V \sqrt{T_{A/D}}$$

- Comments:
  - SNR is proportional to Volume (*V*)
  - SNR is proportional to magnetization ( $m_0 \propto B_0$ )
  - Better SNR for longer acquisitions ( $T_{A/D}$ )

# Resolution Penalty

- Suppose we wished to double the spatial resolution:  
from  $3 \times 3 \times 5 \text{ mm}^3$  to  $1.5 \times 1.5 \times 2.5 \text{ mm}^3$ 
  - Voxel volume decreases by a factor of 8

$$SNR \propto V \sqrt{T_{A/D}}$$

- Based on this expression,  $T_{A/D}$  must increase by  $8^2 = 64$  in order to maintain the same SNR
  - The number of averages might have to increase by 20-30 fold to get  $T_{A/D}$  to 64

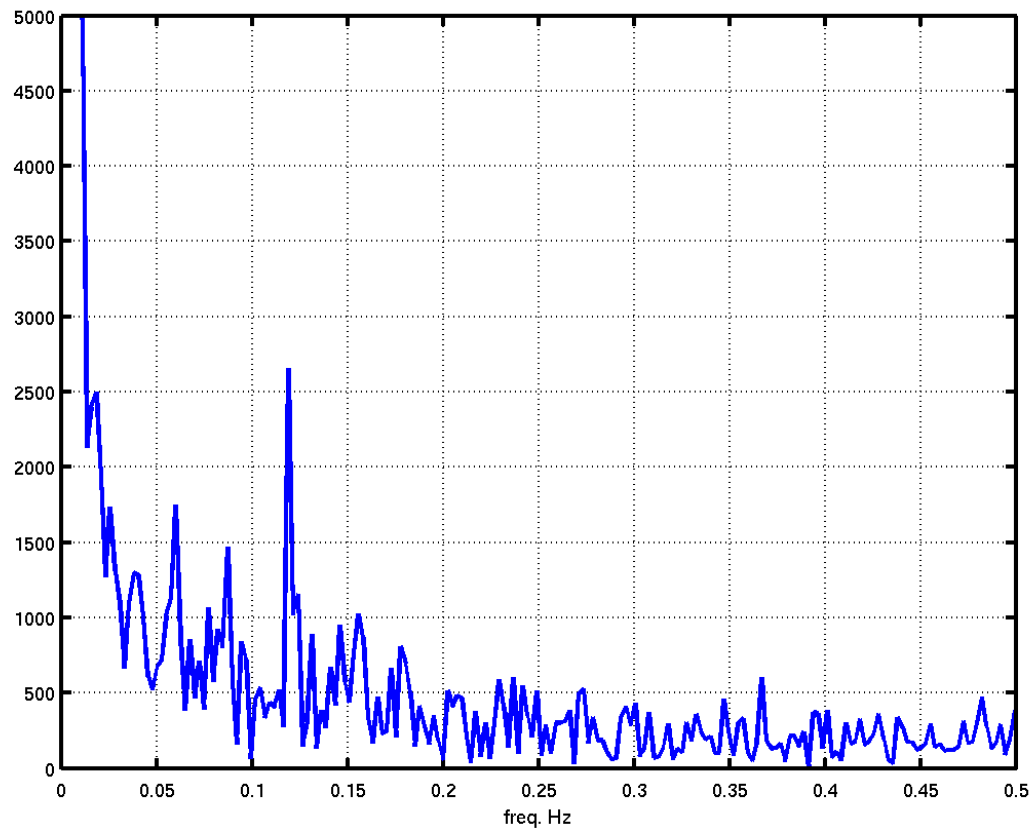
# Image SNR vs. Temporal SNR

- There are two main kinds of SNR that we look at :
  - **Image SNR** – dominated by thermal noise
  - **Temporal SNR** – includes thermal noise, but also includes temporal fluctuations (respiration, cardiac, drifts, trends, equip instabilities, etc., etc.)
- Temporal SNR is most important for fMRI
  - We detect task related signal changes over time

# “1/f” noise and drift

- Temporal Noise in FMRI is typically thermal white noise plus “1/f” noise.
- 1/f noise is more cumbersome (also more interesting?)
  - General Linear Model solvers assume independence
  - 1/f noise means autocorrelation (dependence)
- Contributing sources:
  - equipment instability (heating)
  - Physiological fluctuations
  - temperature drift
  - Neuronal changes

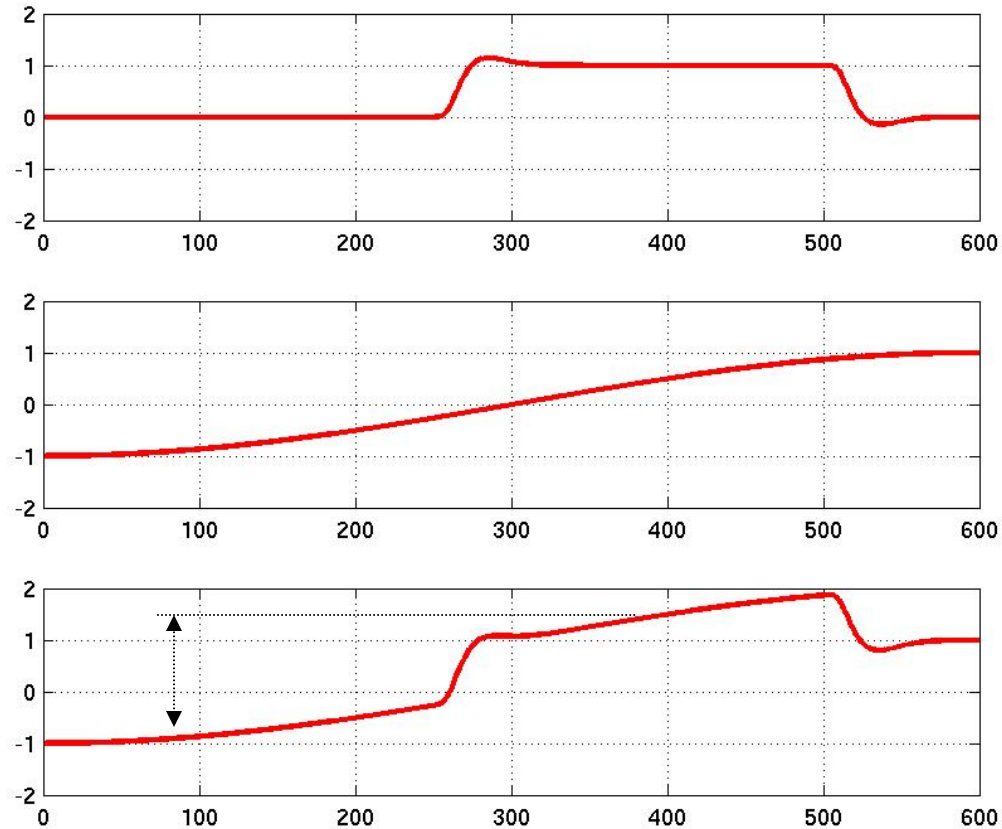
# Drift and 1/f noise



# Why do we care so much about 1/f noise?

- Slow paradigms: Activation is **CONFOUNDED** by 1/f effects.
  - drug effects
  - basal state
  - session effects
  - ...
- Friston, K., Josephs, O., Zarahn, E., Holmes, A., Rouquette, S., and Poline, J.-B. (2000). To smooth or not to smooth? *NeuroImage*, 12:196-208.
- Wang J. **Aguirre** GK. Kimberg DY. Roc AC. Li L. Detre JA. Arterial spin labeling perfusion fMRI with very low task frequency. *Magnetic Resonance in Medicine*. 49(5):796-802, 2003 May.

# Slow drifts as confounds



# Fixes to $1/f$

- Make the task design at higher frequency and filter lower frequencies out.
  - Use something else:
    - T2\* mapping
    - ASL
    - Others ... (VASO)
- both use pairs of adjacent images and subtract out the drift.
- Model the drift with an Auto-regressive model and remove it as a confound. (more on this next week)



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  - Image distortions (susceptibility, ghosting)

# Spatial Normalization: Correcting for Morphological “Noise”

- Morphology varies over a lot over subjects
- Additionally, different brains may be organized differently (e.g- language, handedness...)
- We have to work under the assumption that brains are “close enough” to each other.
- Maybe there is some sort of transformation that we can do to a brain to make it match some “canonical” brain.
- This transformation is not “rigid-body” (ie- varies over the object - produces warp)

# Spatial Normalization: Correcting for Morphological “Noise”

- Warp all subjects' images so that they match a template (*canonical* brain) that is the “paragon of braininess” .
  - MNI templates
  - Talairach and Tournoux
  - Many other specialized ones ...
- Main approaches to warping:
  - higher order affine transformations that include skewing terms
  - deformation fields (non-linear warping) .We' ll focus on this latter one.

# Non-linear warping

- Objective: find a transformation that will minimize the difference (Error) between the template and the object.

$$Error = \sum_{pixels} (I_1 - I_2)^2 + R$$

*(note we could also use the same cost functions as with realignment - correlations, MI, ...)*

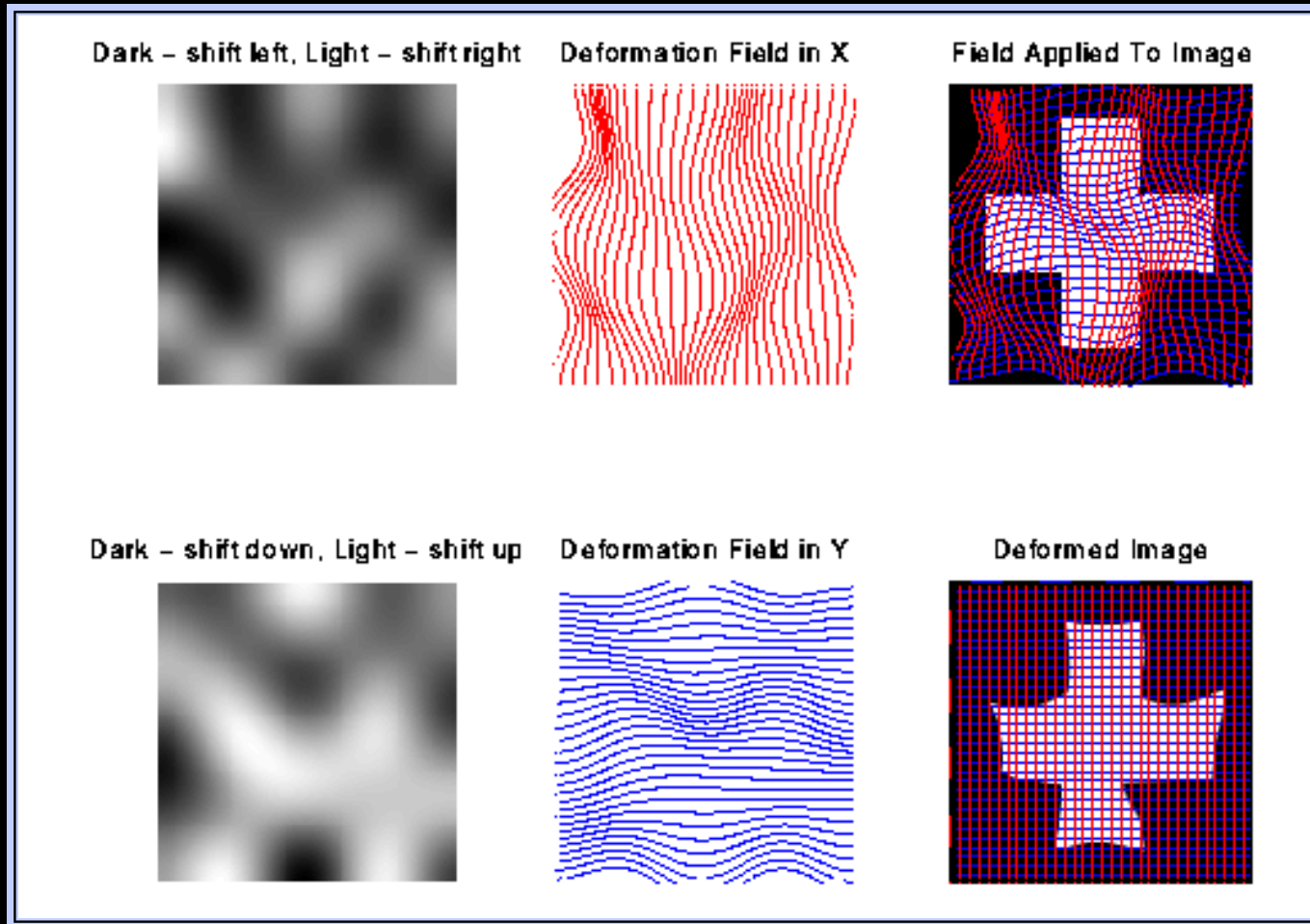
*R is a “regularization” term, typically a spatial derivative to penalize roughness.*

# Non-linear warping

- Different from “Rigid Body” transformations
- Let the transformation be made up of a *different shift at each location*.
- Assume this amount of shift (warp) is a smooth and continuous function over the 3D space we’re working on. Let’s call it

$$s = W ( r )$$

# Sample deformation field



# Non-linear warping

- Approximate the *warp()* function as a series. Could be Taylor series, Fourier, Euler, ...etc. (It turns out that Discrete Cosine Transforms are particularly good for this application.)

$$W(\mathbf{r}) = \sum_i a_i \cos(\omega_i \mathbf{r})$$

- Find the first few coefficients  $a_i$  to approximate the *warp()* function in *each* direction (x,y,z shift).

(This means that there are three, 3D, warping functions to find)

# A SERIES of unfortunate ...

Everything is  
a sum of  
polynomials

This tea  
is cold...



Brook Taylor  
1715

Everything is  
a sum of  
sinusoids

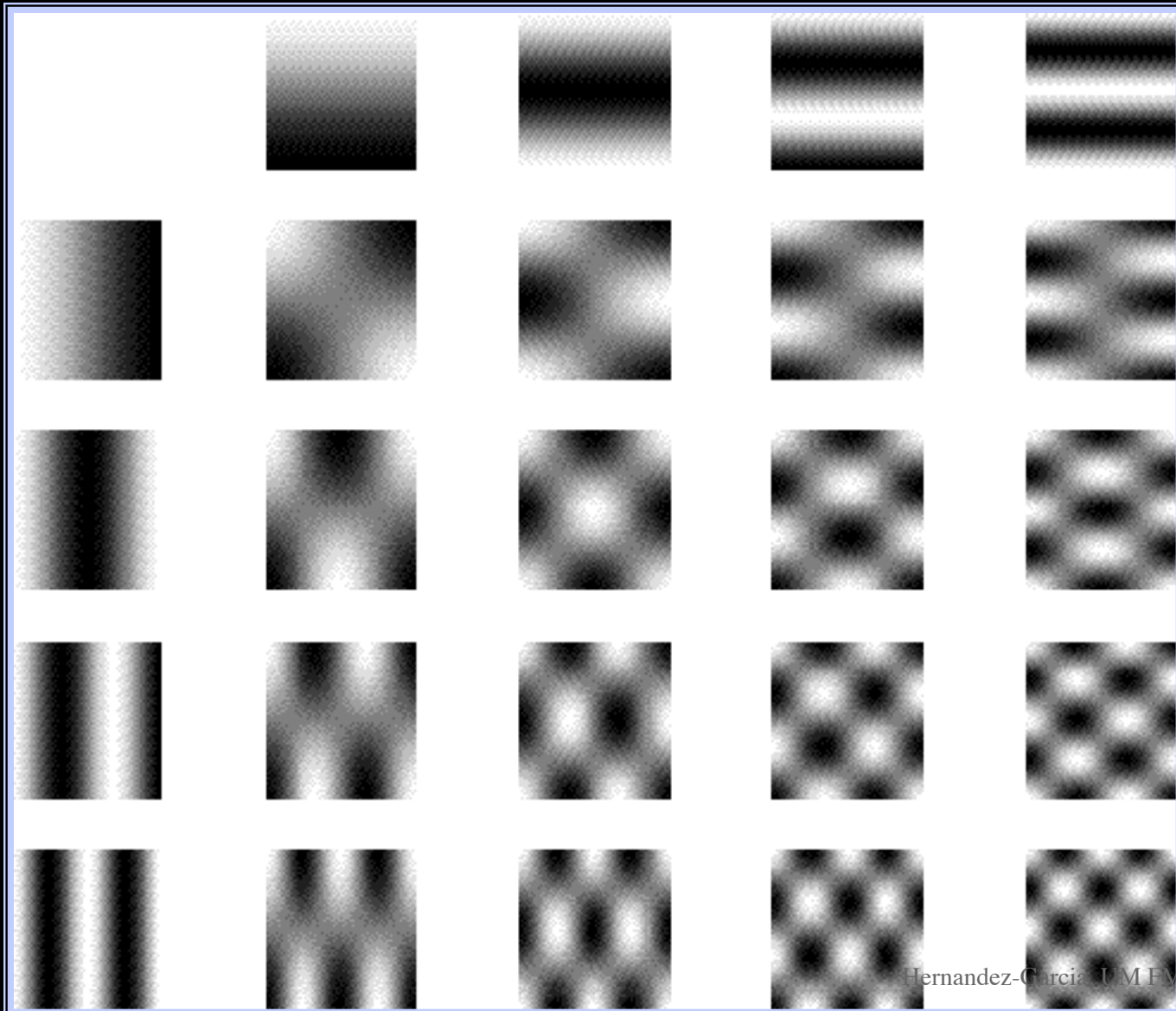
I hope they  
don't chop off  
my head ...



Jean-Baptiste  
Joseph Fourier



# Basis Functions



# Non-linear warping

- Finding the coefficients is again an optimization problem...
- Strategies: least squares, Gauss-Newton, simplex, gradient descent, genetic algorithms, neural networks
- ... Just minimize the cost function.

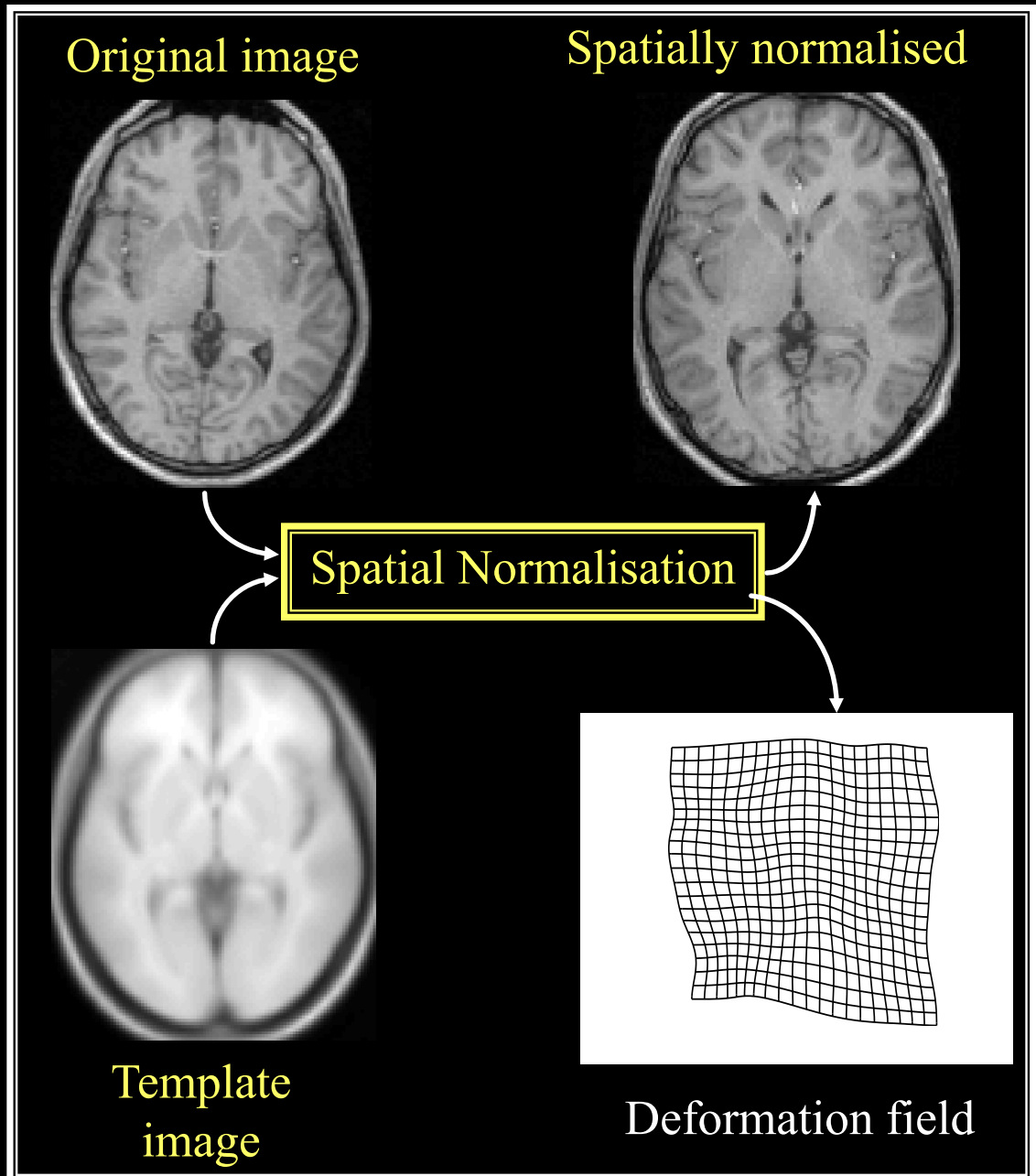
# Spatial Normalisation

Determine the spatial transformation that **minimises the sum of squared difference** between an image and a linear combination of one or more templates.

Begins with an **affine registration** to match the size and position of the image.

Followed by a **global non-linear warping** to match the overall brain shape.

Uses a Bayesian framework to simultaneously **maximise the smoothness of the warps**.



# Normalization

## References

Friston et al (1995): *Spatial registration and normalisation of images.*

Human Brain Mapping 3(3):165-189

Ashburner & Friston (1997): *Multimodal image coregistration and partitioning - a unified framework.*

NeuroImage 6(3):209-217

Collignon et al (1995): *Automated multi-modality image registration based on information theory.*

IPMI' 95 pp 263-274

Ashburner et al (1997): *Incorporating prior knowledge into image registration.*

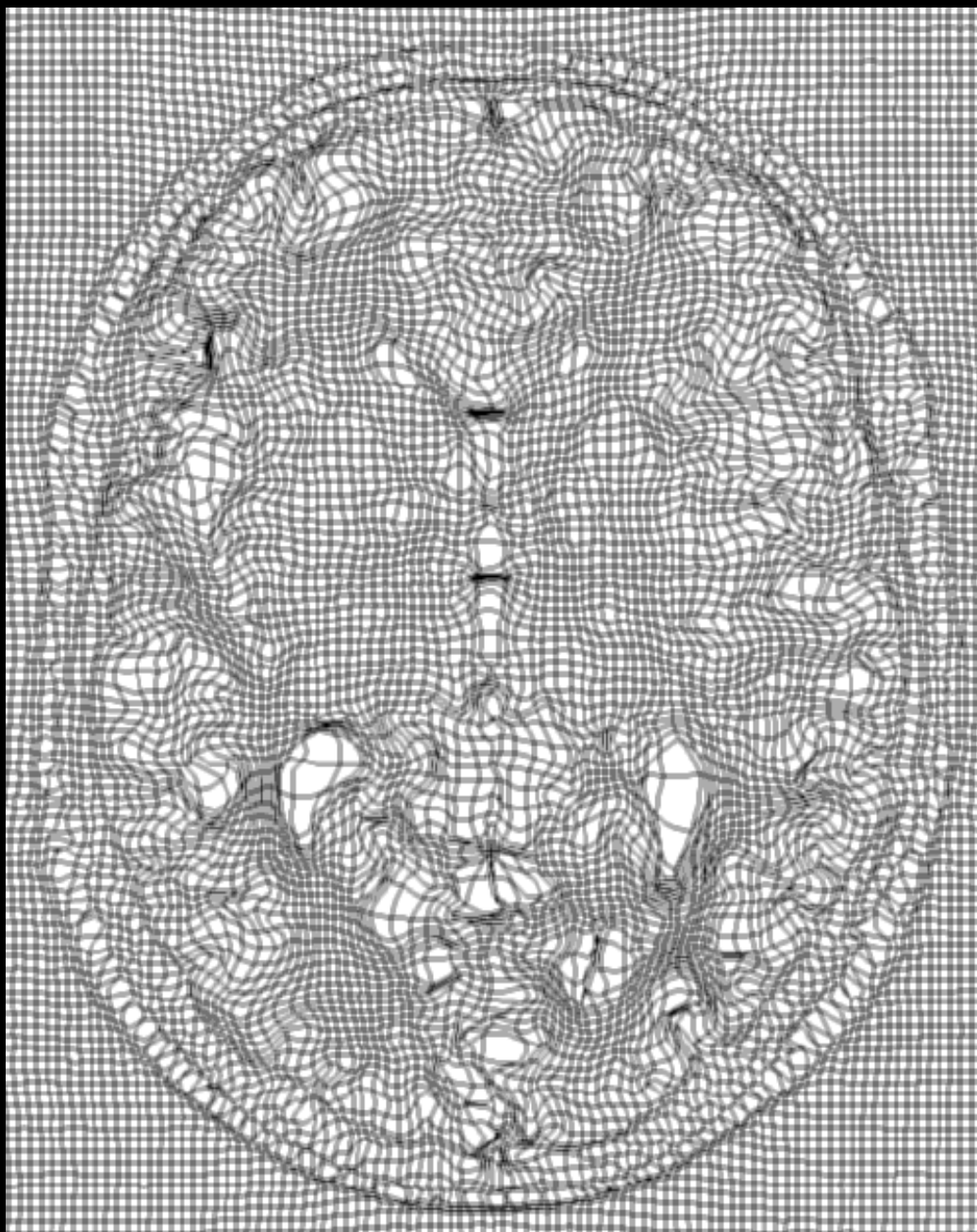
NeuroImage 6(4):344-352

Ashburner et al (1999): *Nonlinear spatial normalisation using basis functions.*

Human Brain Mapping 7(4):254-266

Ashburner & Friston (2000): *Voxel-based morphometry - the methods.*

NeuroImage 11:805-821

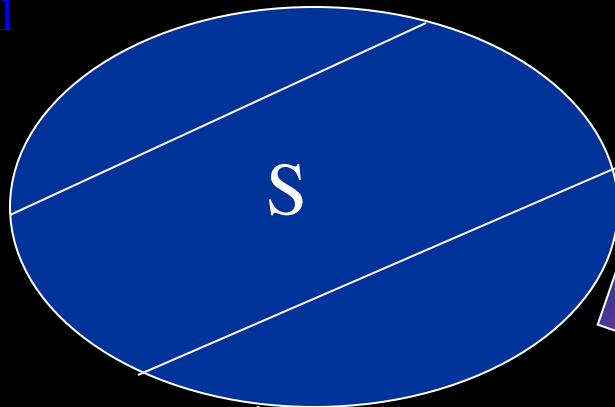


# Practical aspects of spatial normalization

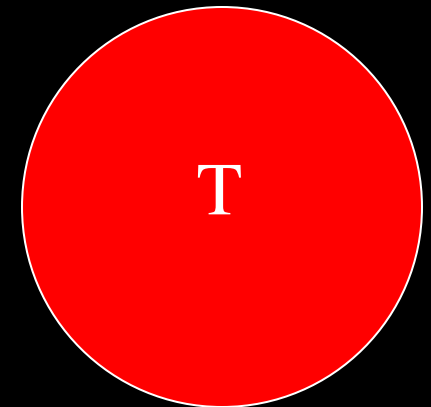
- Resolution , contrast , Field Of View can be different between template and functional images.
- Useful to collect additional images to use in the search for the deformation fields, then apply the resulting deformations to the functional images.

# Practical Normalization Path: Moving between 3 frames of reference

Whole head  
image  
(hi-res , T1  
weighted)



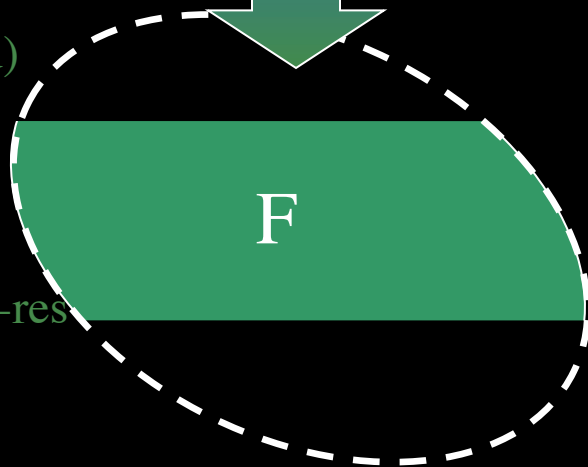
non-linear  $W_1$



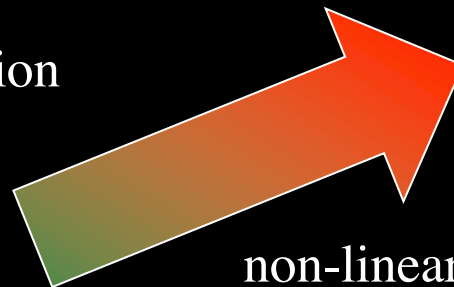
Template  
(med-res , T1  
weighted)

T1-weighted  
overlays (hi-  
res , T1  
weighted)

linear  
transformation



non-linear  $W_2()$   
(sometimes  
not enough info...)



functional  
images (hi-res  
, T2\*  
weighted)

# Typical Normalization path

1. Register whole brain to overlay.

$$e = (F - AS)^2, \text{ find } A \text{ that minimizes } e$$

2. Warp the transformed whole brain image into template

$$e = (T - W_2(AS))^2, \text{ find } W_2 \text{ that minimizes } e$$

3. Use the same warping parameters to warp the functional maps. The end result is:

$$W_2F$$

S = whole brain image

F = functional image

T = template image

# Segmented Normalization

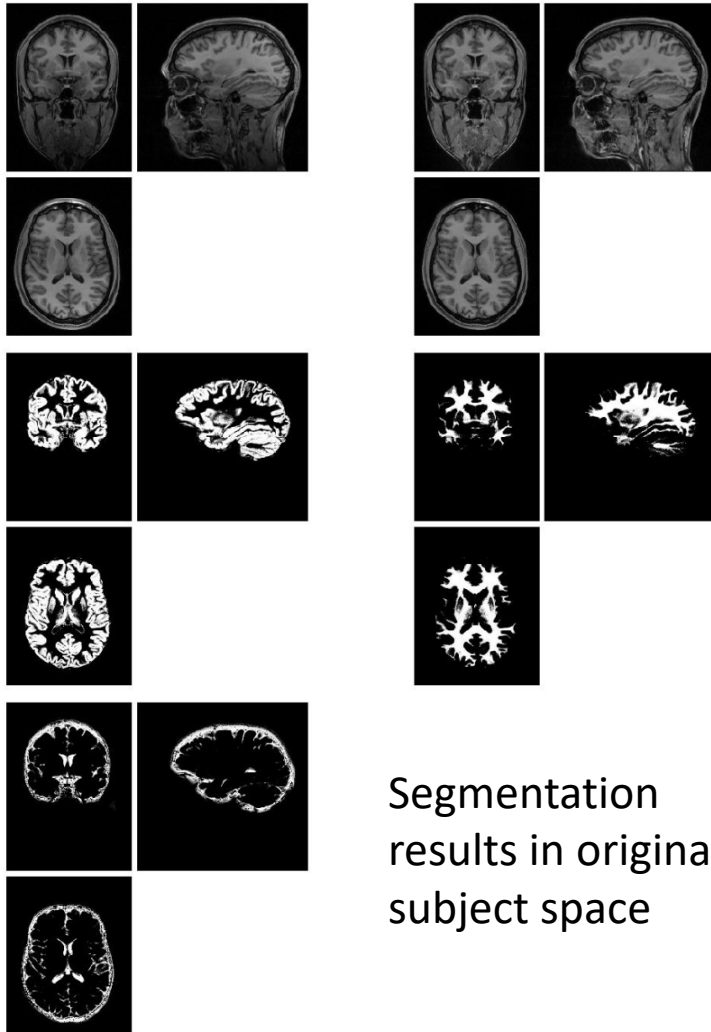
- Warping and Normalization algorithms have evolved considerably. Many variants and strategies motivated by morphometry studies (e.g. Brain Voyager).
- Knowledge of brain structure (e.g. grey matter, white matter, CSF) can improve the normalization process
- Strategy: partition the brain into GM, WM, and CSF and then performs a more “informed” normalization on the resulting partitions



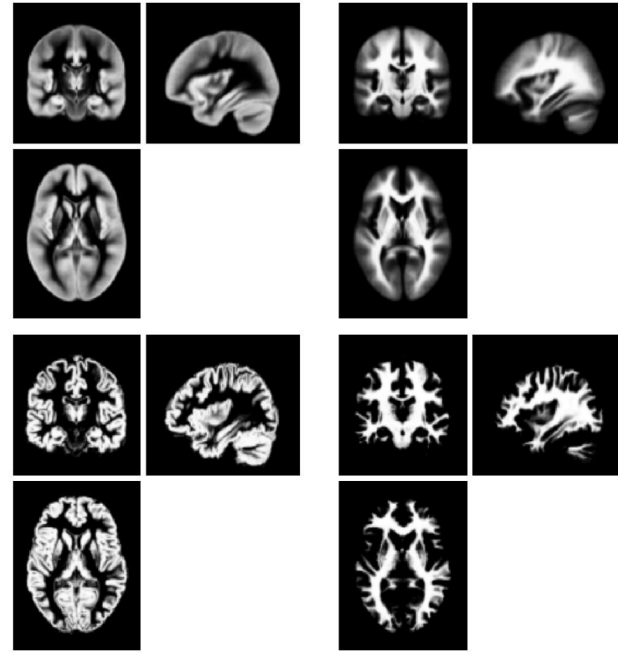
# Segmented Normalization

1. Use clustering algorithm to calculate intensity distributions of grey matter, white matter, and CSF (additional clusters for eyes and scalp too)
2. Normalize resultant segments to template
3. Significant improvements in normalization

# Segmented Normalization



Segmentation results in original subject space



Tissue probability maps

Normalized tissue maps

# Lecture Goals

- Understand the following confounds in fMRI and what corrections exist
  - Slice Timing effects (temporal shifting)
  - Movement (rigid body realignment)
  - Physiological Artifacts: respiration and heart beat (regression filters)
  - Electronic Noise (filters and autoregressive models)
  - Morphology (non-linear warping)
  - Image distortions (susceptibility, ghosting, warping)

# When things actually go wrong

- Up to now we have explored the charming features of the fMRI signals... They are expected to be there.
- Now we'll look at a whole other set of complications that are not present in a consistent way.

# Unwanted Things that can happen to the MR signal

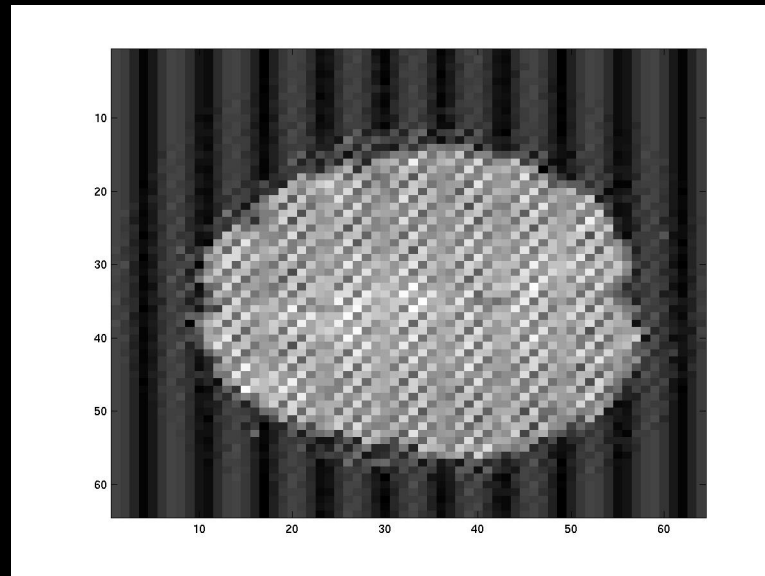
- Addition of “junk”
  - Coherent at different frequencies
  - Incoherent (spikes)
- Multiplication (modulation) by “junk”
  - Unwanted frequency shifts
  - Unwanted Phase shifts

What do these do to the image?  
(Think Fourier Transform)

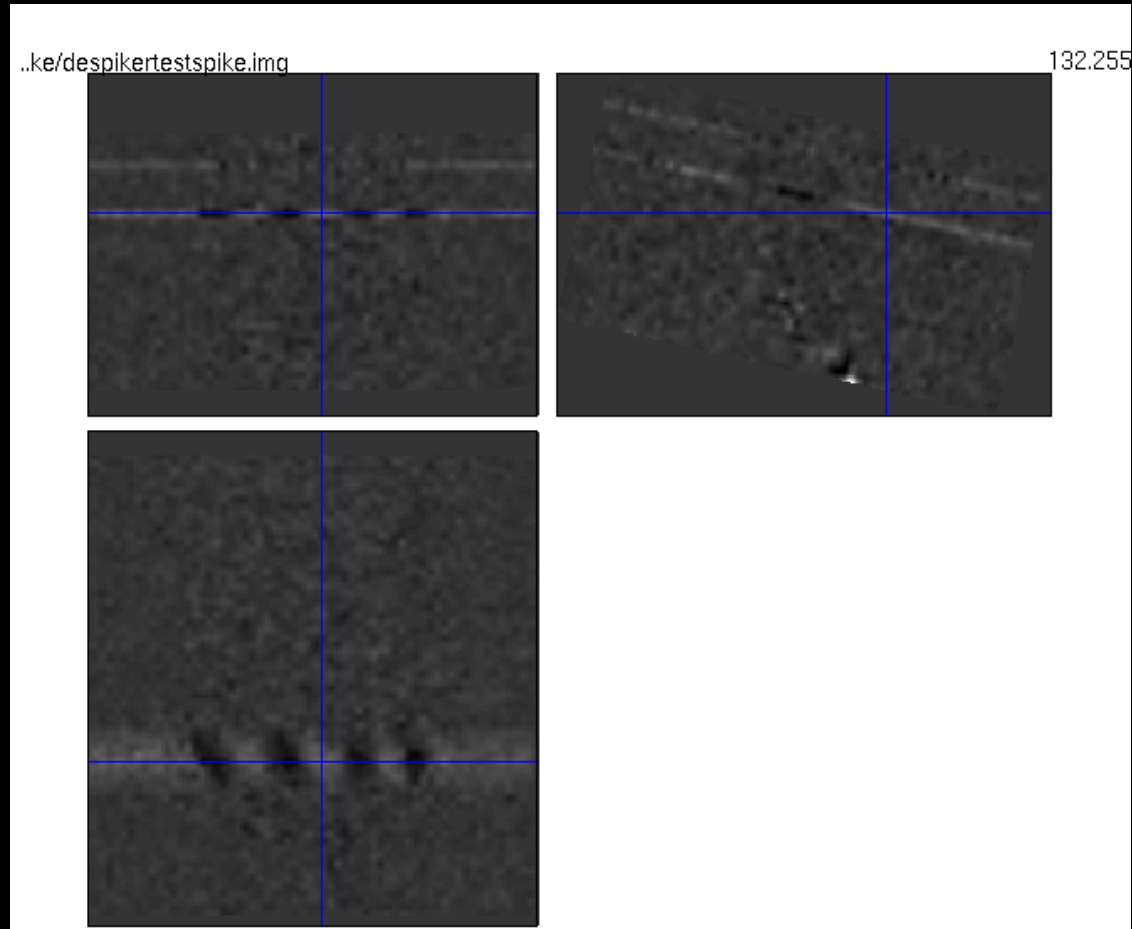
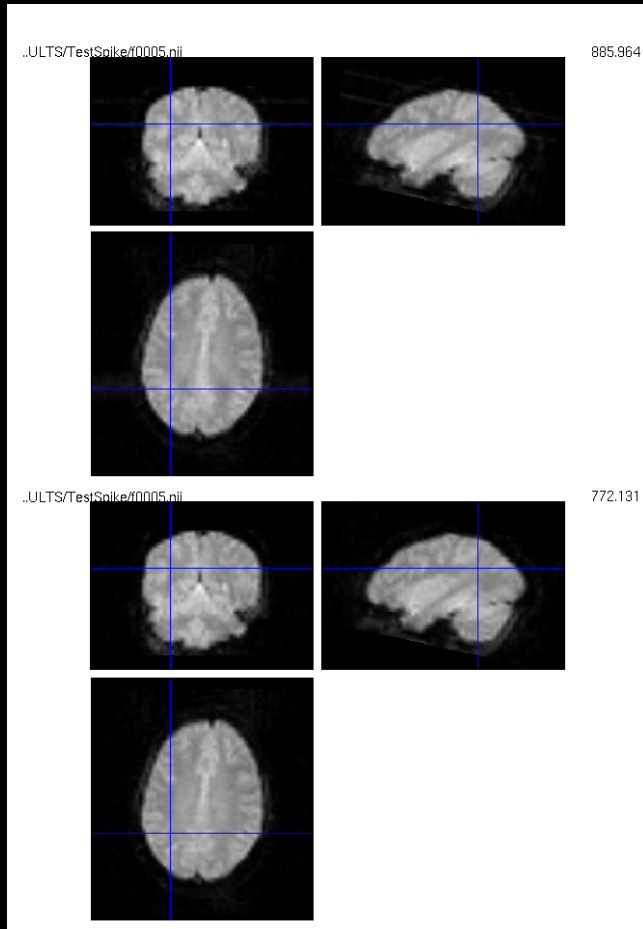
# White Pixel Artifact

(added junk)

- Caused by a noise spike during acquisition
- Spike in K-space  $\leftrightarrow$  sinusoid in image space



# Not Always Easy to See...

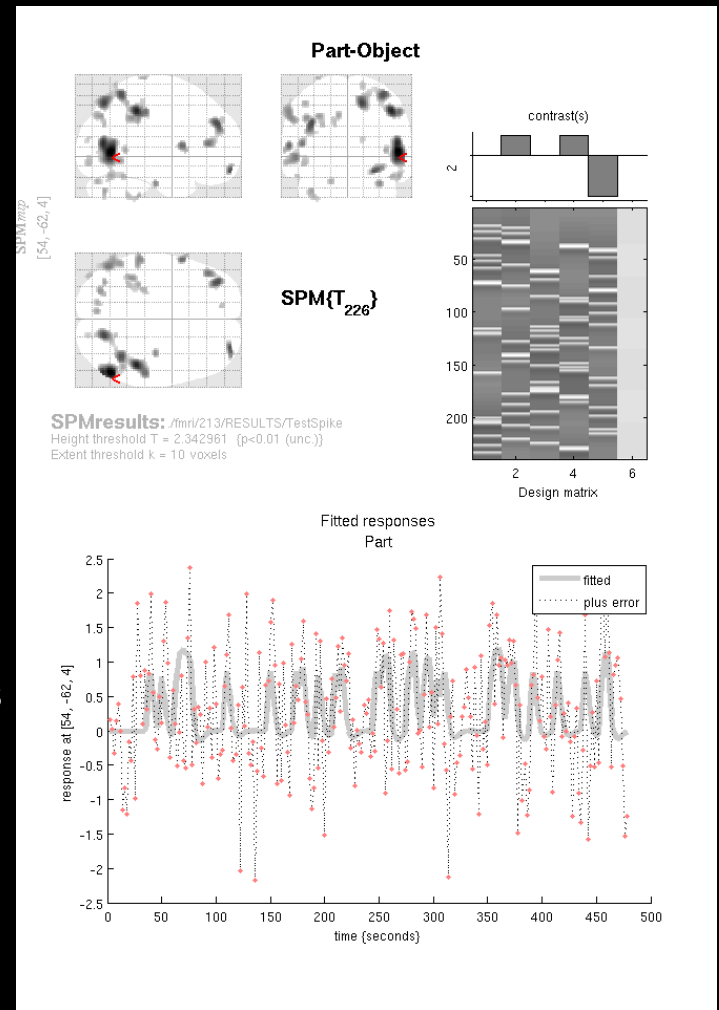
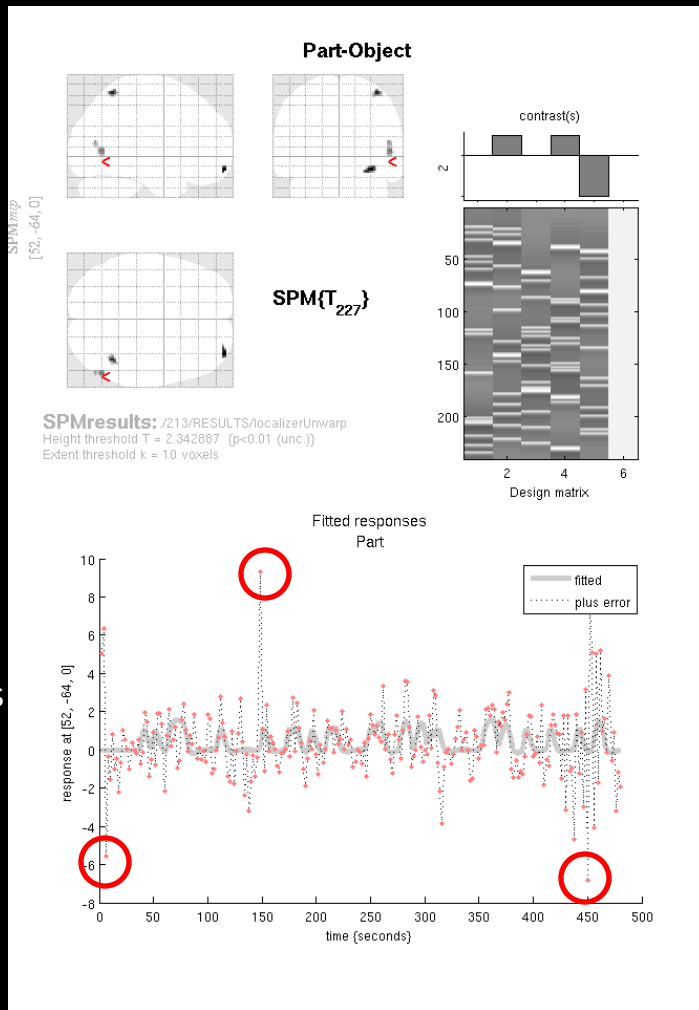


Top image has spikes, bottom does not

Difference of the two images

Hernandez-Garcia, UM FMRI course

# Spikes In Results - Corrected



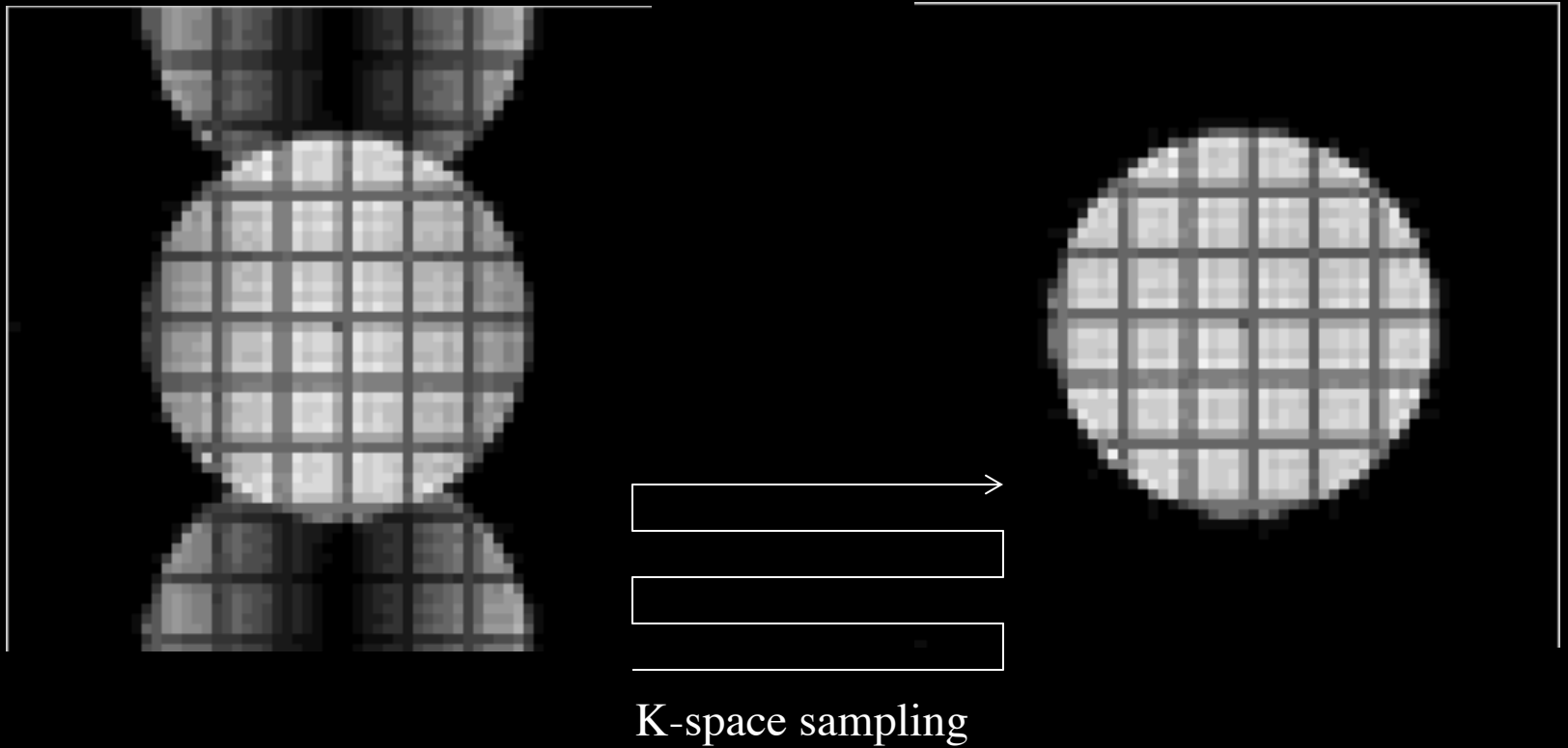


# Despiking

- How do I know if I have a problem?
  - Look for large changes in global signal
  - Difference images to make spikes more visible
  - Look for large deviations from predicted response
- How do I fix it?
  - Treat the artifacts as early as possible, either in k-space or in voxel-space before other preprocessing steps have been applied
  - Replace spike with interpolation of neighbors

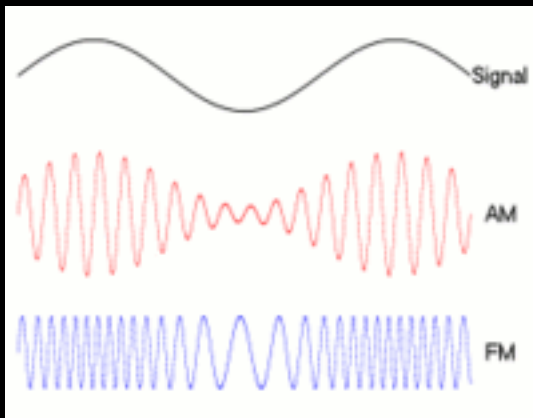
# Ghosting

(modulation by junk)



# Modulation

(...just when you thought you were done with high school trigonometry!)



$$\sin \alpha \cos \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

$$\cos \alpha \cos \beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

$$\sin \alpha \sin \beta = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$

$$f(t) \sin \omega_0 t \xleftrightarrow{\mathcal{F}} \frac{i}{2} [F(\omega + \omega_0) - F(\omega - \omega_0)]$$

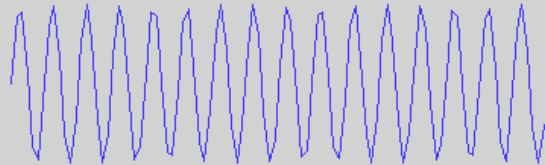
# Modulation

(...just when you thought you were done with high school trigonometry!)

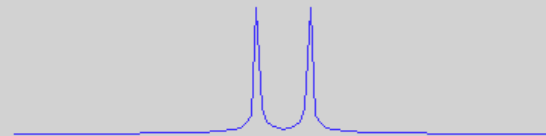
Time domain

Frequency Domain

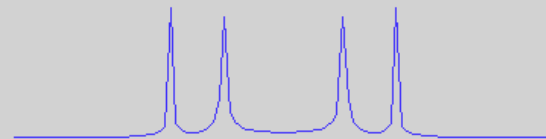
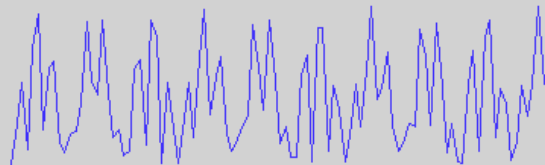
$A(t)$



$B(t)$

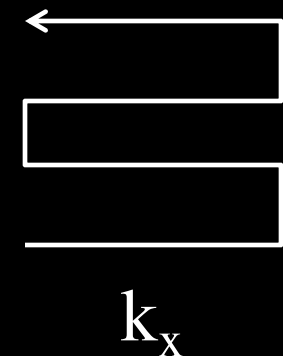


$A(t) * B(t)$

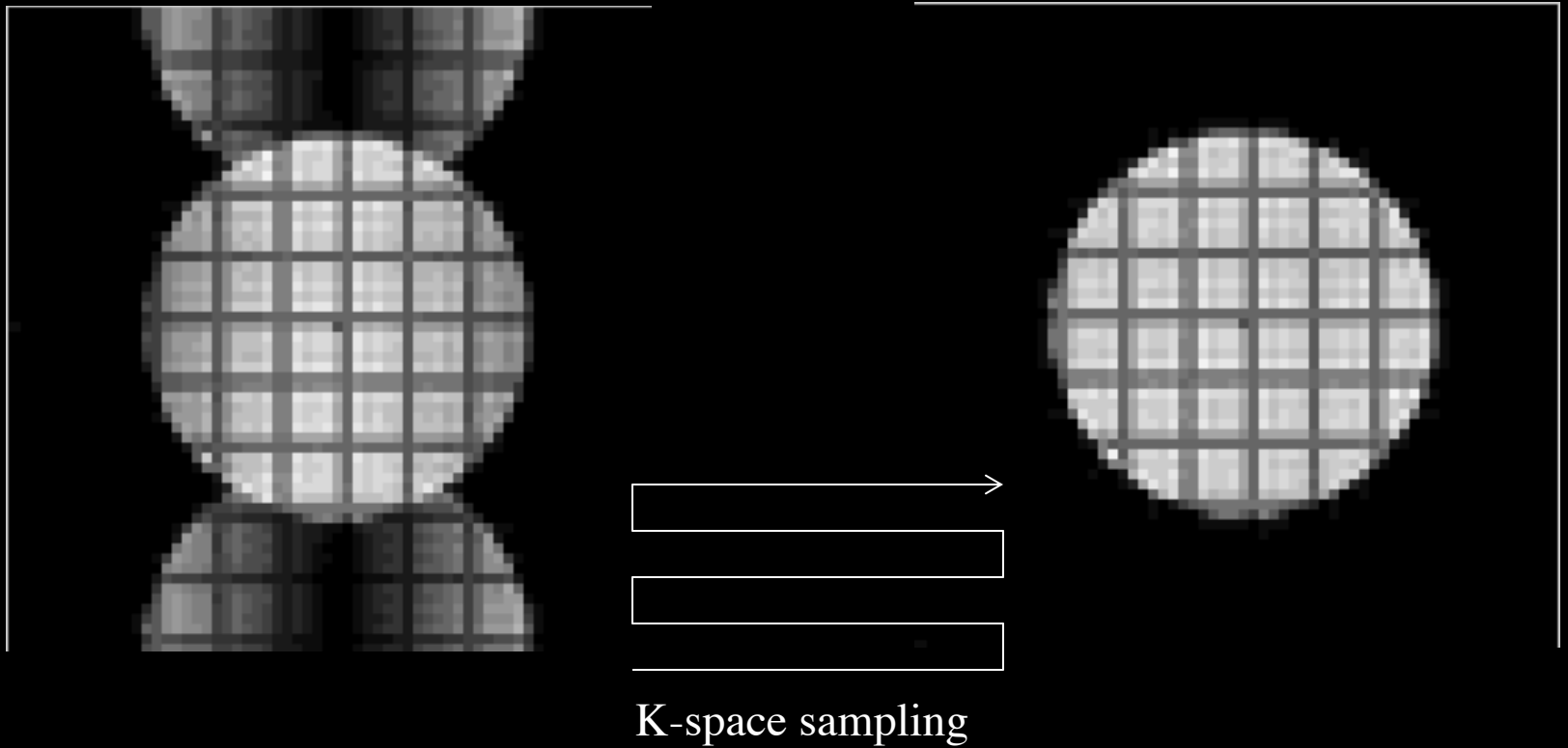


# EPI Nyquist ghost

- Caused by phase-error every other line of k-space (hardware problem - e.g.- sometimes the gradient coils are not well balanced)
- This means k-space data are **modulated along one axis** by artefact
- Artefact is **oscillation** at the Nyquist frequency.
- Solution can be easy:
  1. add a little bit of phase to alternate lines of k-space and reconstruct.
  2. See if the ghosting gets better or worse.
  3. Repeat until fixed.



# EPI Nyquist Ghost



# Off-resonance effects

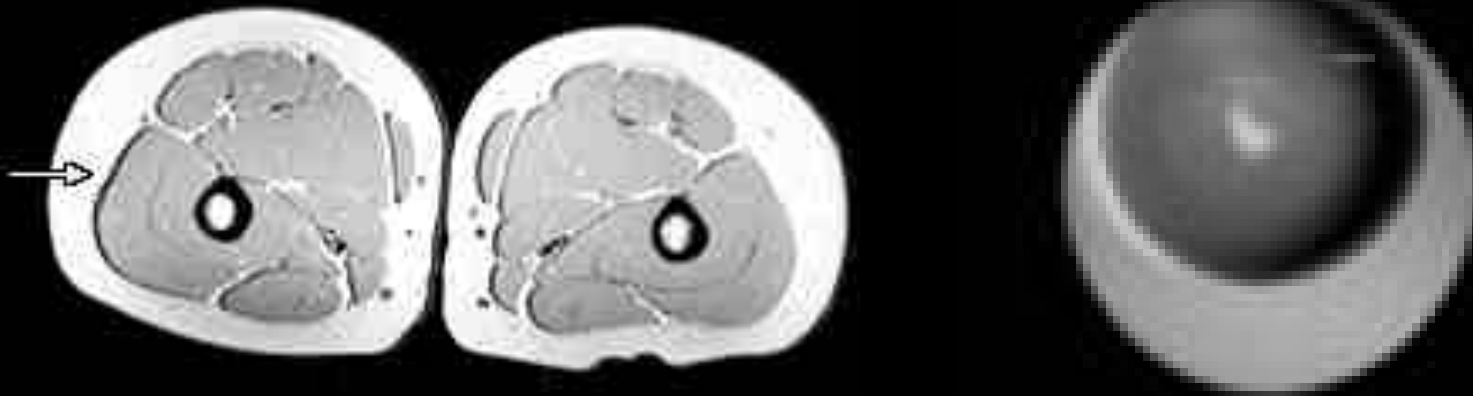
(frequency shifts)

- Recall:

$$\omega_0 = \gamma B_0$$

- **Chemical shift:** fat protons have a different gyromagnetic ratio, and hence resonant frequency (3.5ppm away from water)
- **Field distortions:** produce the same effect (changes in  $B_0$  instead of change in  $\gamma$ ). Both result in a change of  $\omega_0$  in a particular region

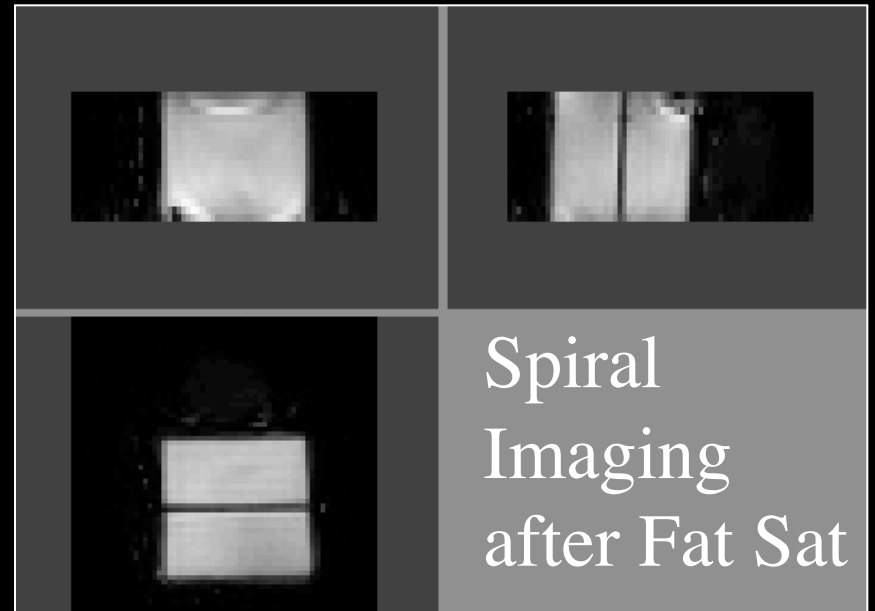
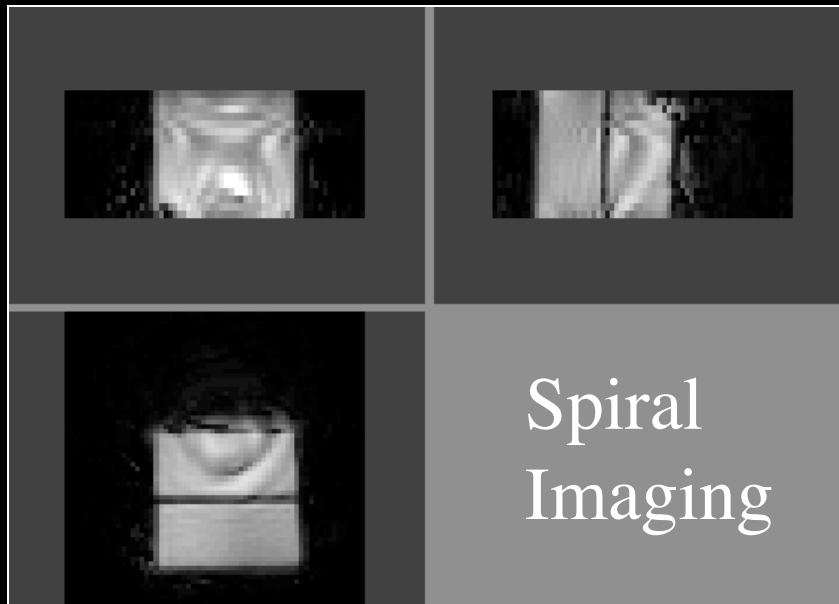
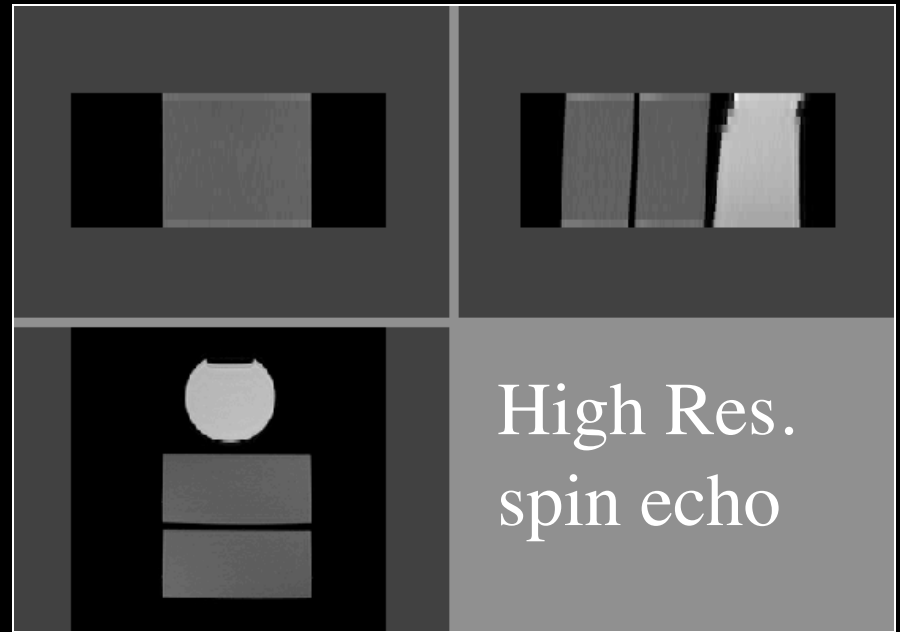
# Examples of Chemical Shift Artifact



Images from: Hornak, <http://www.cis.rit.edu/htbooks/mri/chap-11/chap-11.htm>



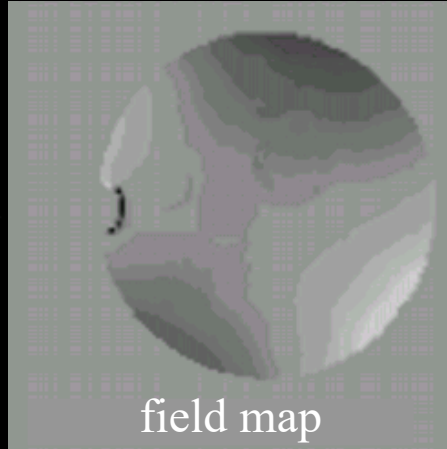
# Chemical Shift Artifact (spiral imaging example)



# Geometric Distortions



spin echo image

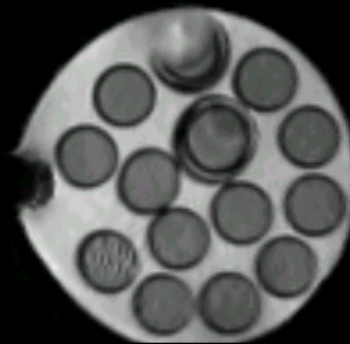


field map

Jezzard and Balaban, MRM 34:65-73 1995



warped epi image



unwarped epi image

# Geometric Distortion

- Caused by Bad “shim” and/or non-linear gradients.
  - The gradient you want is not always the gradient you get.
- Solutions:
  1. Correct using field maps.
    1. Measure  $B_0$  map
    2. calculate how much extra phase is due to the inhomogeneity,
    3. remove “bad” phase from data (not easy)
  2. correct by warping the image to match an undistorted one

(NB – These work to a point. Sometimes you can't separate signals that have been pushed together by the artifact:

You can't recover signal from voxels where all the signal is gone completely)

# Distortions are usually “errors” or unexpected terms in the Signal Equation

$$S(t) = \int m(x, y, z) e^{i2\pi(k_x(t)x + k_y(t)y + k_z(t)z)} dx \cdot dy \cdot dz$$

$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau \quad + \text{junk}$$

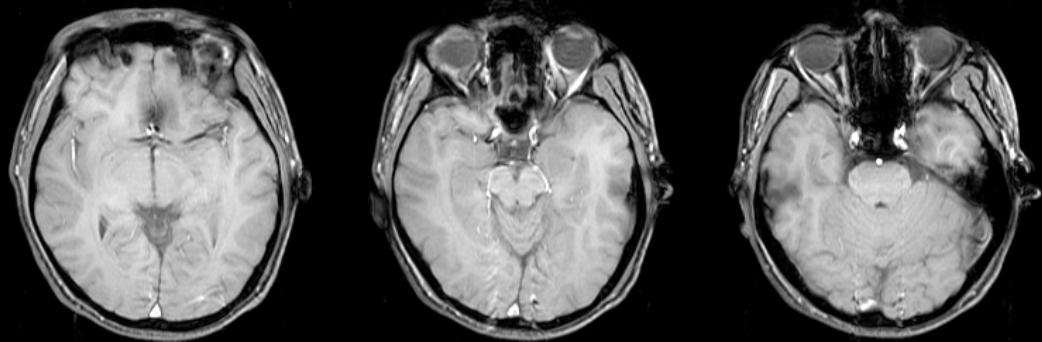
$$k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau \quad + \text{junk}$$

$$k_z(t) = \frac{\gamma}{2\pi} \int_0^t G_z(\tau) d\tau \quad + \text{junk}$$

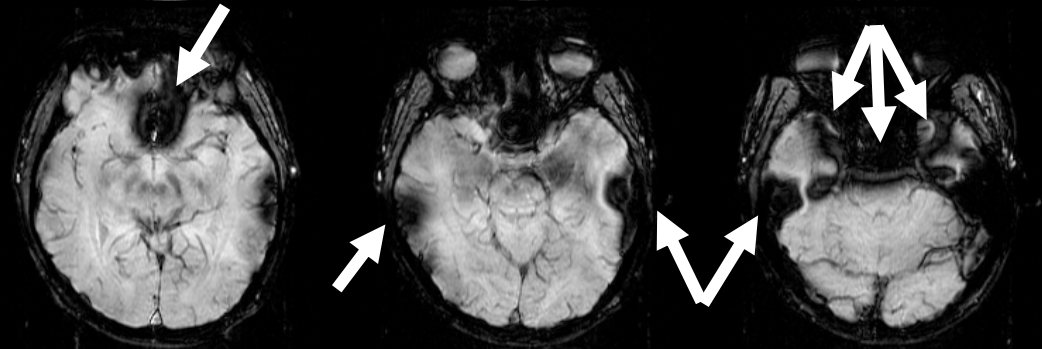
# Susceptibility Artifacts

- Off-resonance artifacts caused by adjacent regions with different magnetic Susceptibility
- **BOLD** signal requires susceptibility weighting... but this also leads to image artifacts

No  
Susceptibility  
Contrast



High  
Susceptibility  
Contrast



# Magnetic Susceptibility

- Amount of Magnetization of a material produced in response to a magnetic field

$$M = \chi H$$

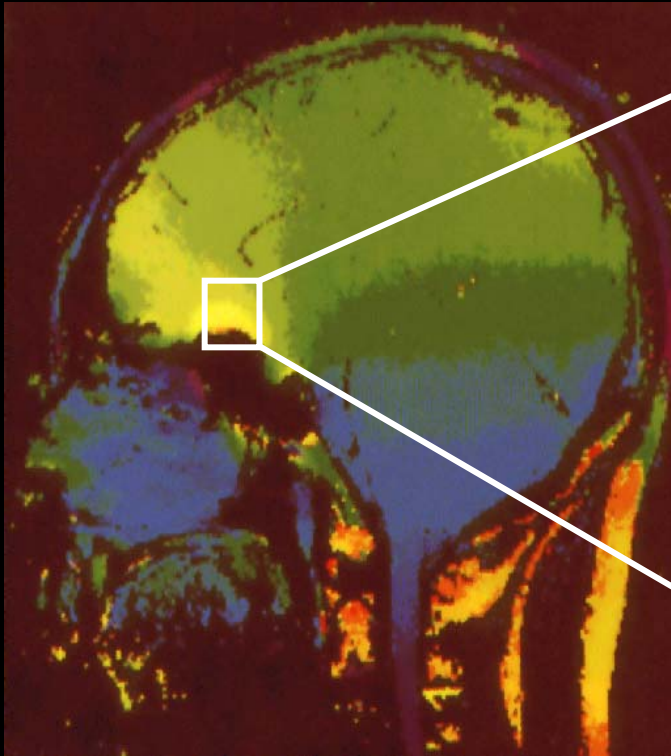
- Field gets distorted by this magnetization

$$B = \mu_0 H + \chi \mu_0 H$$

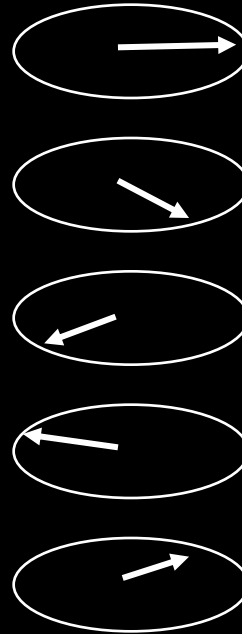
- Gets worse with higher field scanners!

# Susceptibility can produce Signal Loss

Magnetic Fields  
in the Head



Low Field



High Field

Ideal



Signal Loss

Li et al. Magn. Reson. Med. 36:710  
(1996)

courtesy of Douglas Noll

Hernandez-Garcia, UM FMRI course

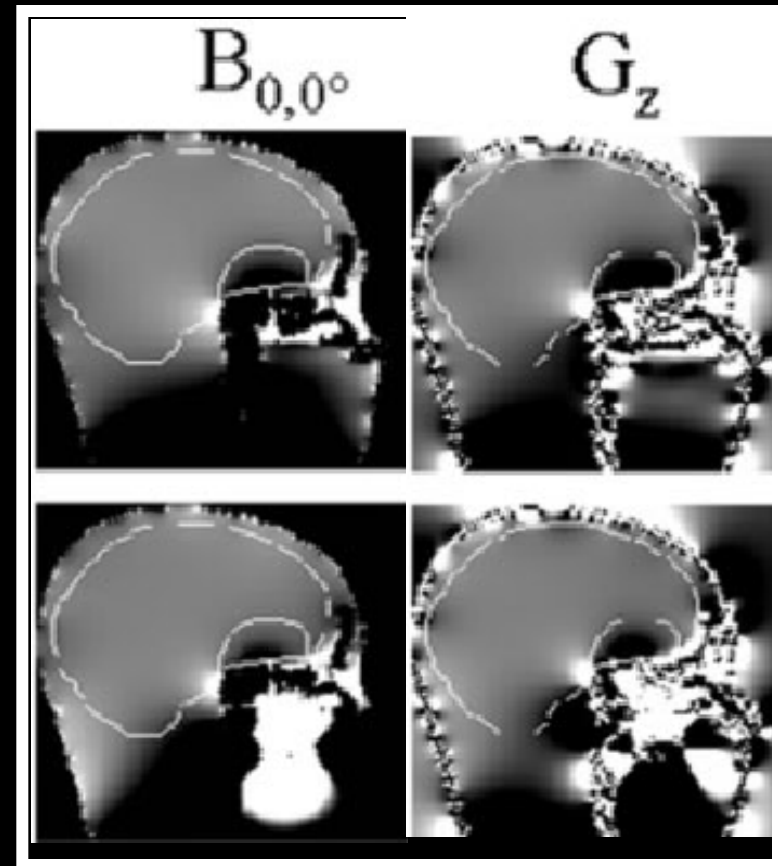
# Susceptibility Artifacts

- Local gradients Challenges:
  - If severe: Lots of different phases within a voxel. Result is destructive interference: signal loss.
  - If they are more gentle: skewing of the k-space trajectory in different voxels)
- Solutions: Lots of tricks you can do, to fix it, but they all have an associated cost in time, SNR, computation, hardware ...
  - Choose acquisition parameters such that the artifacts are minimized ... simplest, usually best!
  - Parallel imaging
  - Z-shimming, active shims
  - Forward-model, iterative reconstructions



# Intra-oral Diamagnetic Shims

- Shimming by 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order shims provides only modest field correction
- Magnetic field can be made more uniform through the use of intraoral shims made of diamagnetic materials



(Wilson & Jezzard P *MRM* 50:1089-94, 2003)

Hernandez-Garcia, UM FMRI course

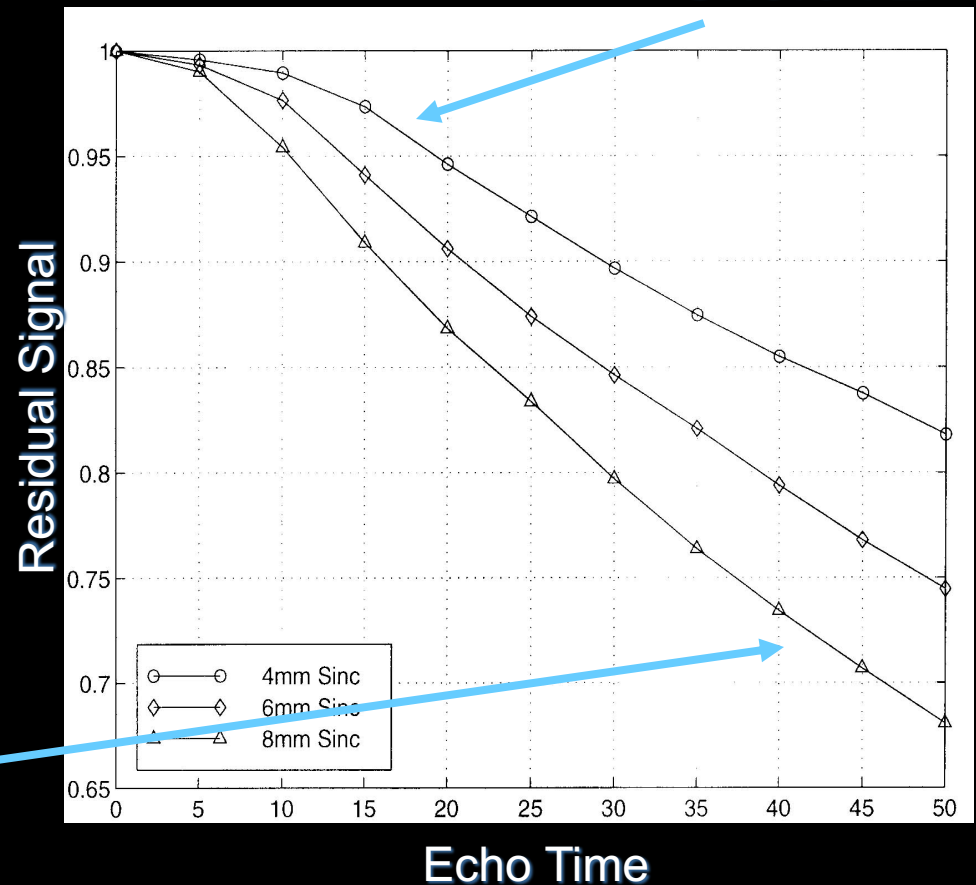
# Why are some images affected by off-resonance, but not others?

- Spin Echoes refocus spins that get out of phase
- Gradient Echoes do not
- Major factors:
  - How much time you allow the effect to accumulate – echo times, readout time
  - How much variation in the magnetic field within the excited volume – slice thickness, shim

# Some Simple Approaches

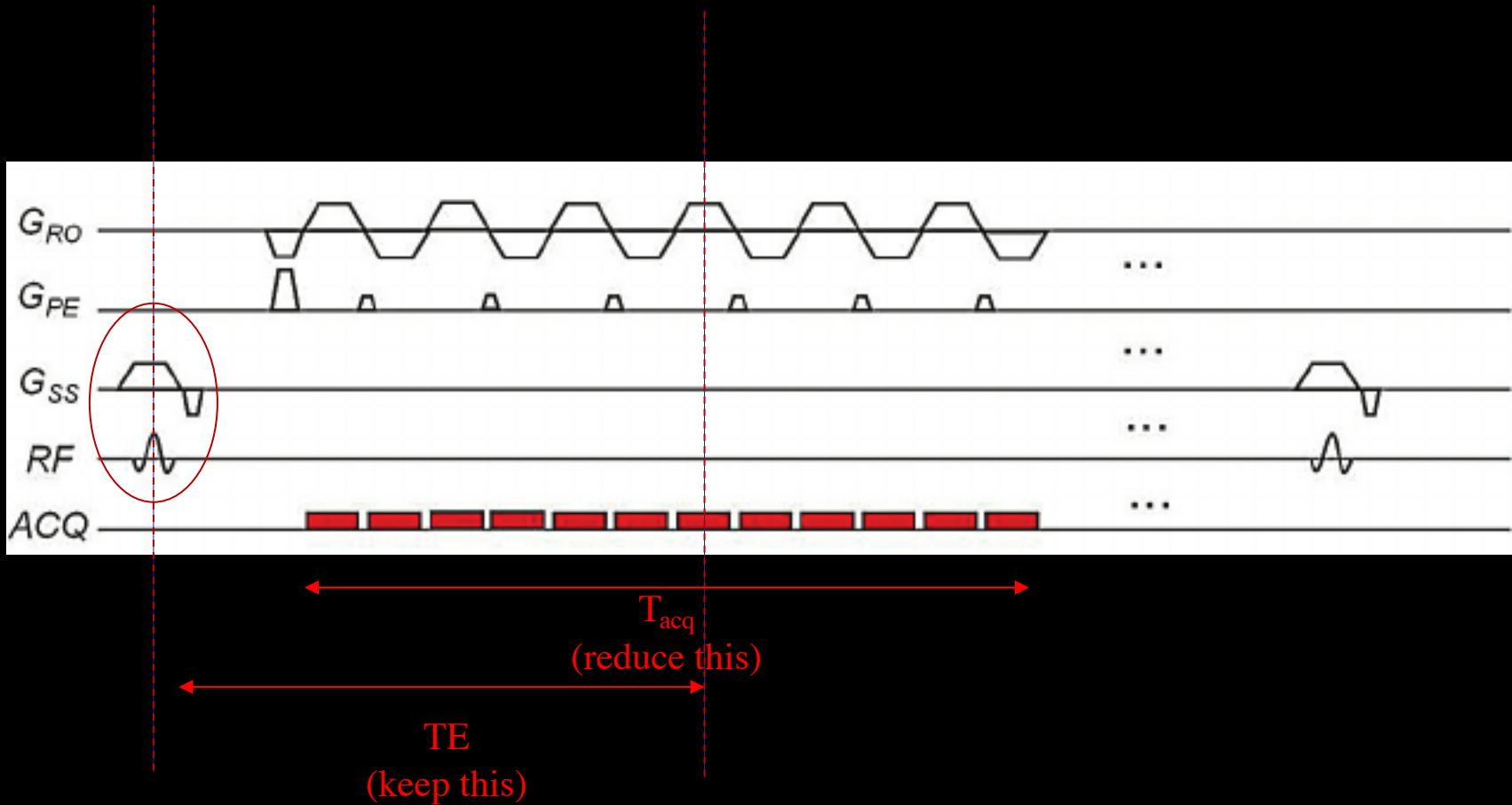
- Thinner slices
  - Slower, more slices to cover head
  - Lower SNR
- Shorter echo time (TE)
  - Reduced contrast to BOLD effect
- Shorter readout !!

Thin Slice  
Short TE

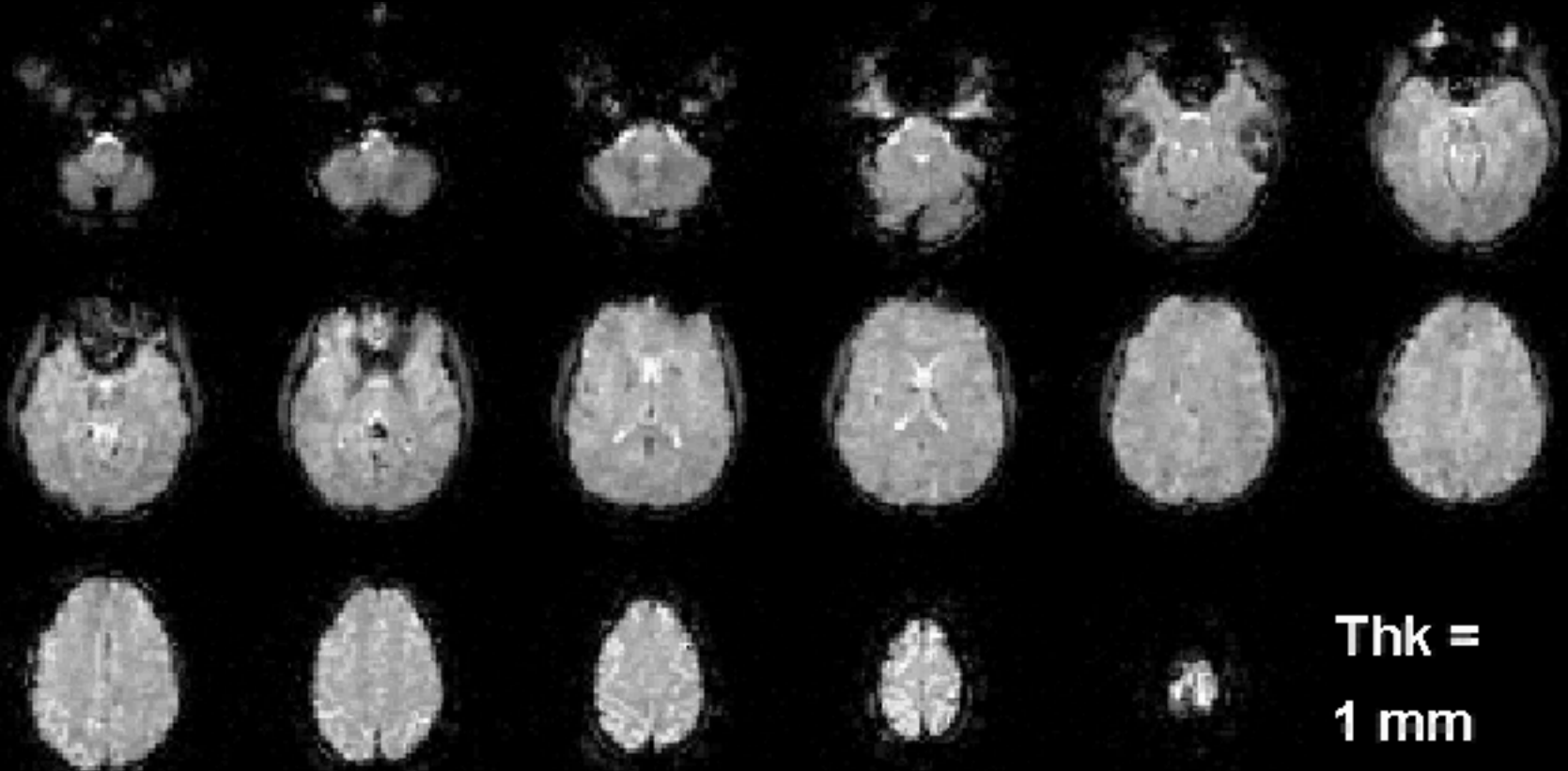


Thick Slice  
Long TE

# EPI pulse sequence

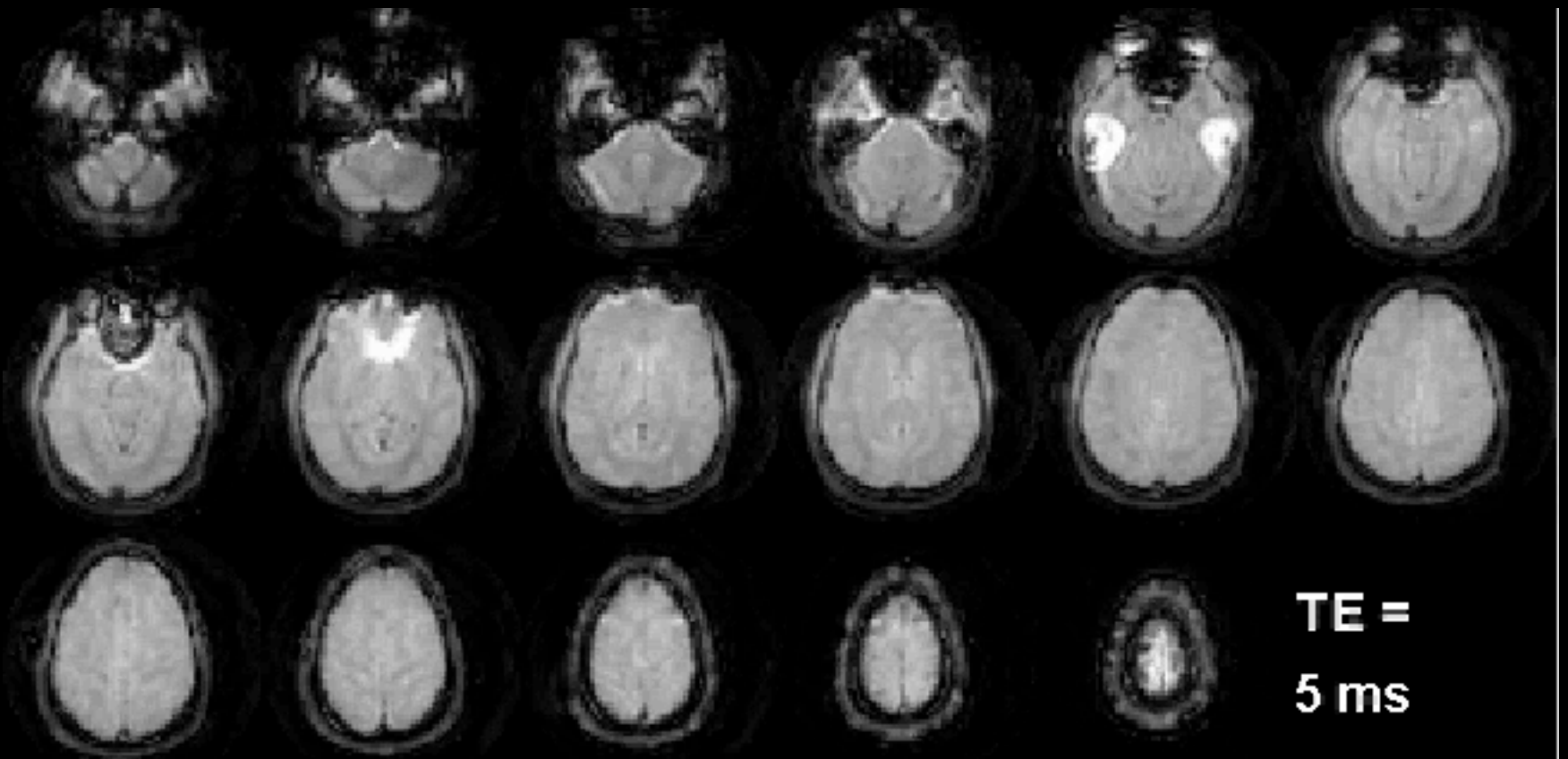


# Signal Loss vs. Slice Thickness (movie)



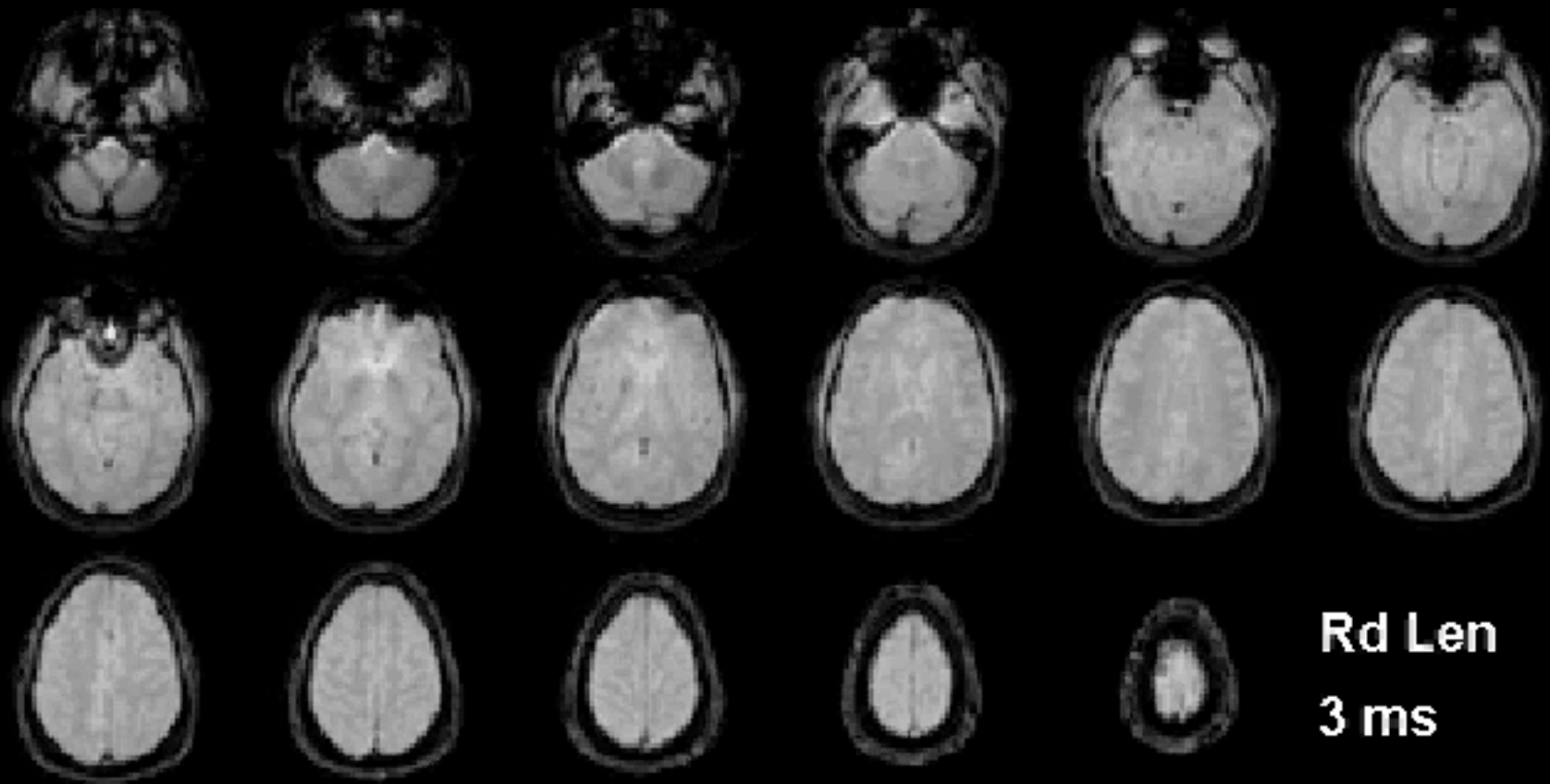
TE = 25 ms, 20 ms Single-Shot Spiral Acquisition

# Signal Loss vs. TE (movie)



Thickness = 4 mm, 20 ms Single-Shot Spiral Acquisition

# Susceptibility Distortions from Long Readouts (movie)



TE = 10 ms, Thickness = 4 mm, Spiral Acquisition

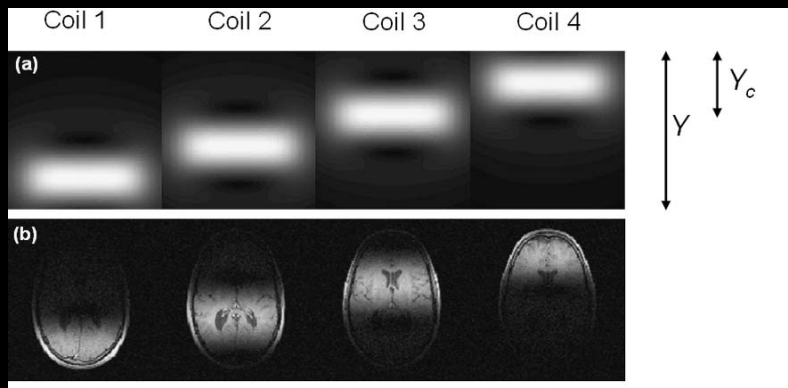
# Reducing Readout Length

- Hardware limits: we can only go so fast!
  - Gradient strength limited by peripheral nerve stimulation
  - Head gradients would be one approach
  - ... but there are still hardware limits!
- Parallel imaging (e.g. SENSE) can reduce readout duration
  - More coils collecting less data per coil.
  - How much data can we skip, and calculate from coil sensitivity?
- Compressed sensing
  - How much data can we skip, and calculate from prior information?



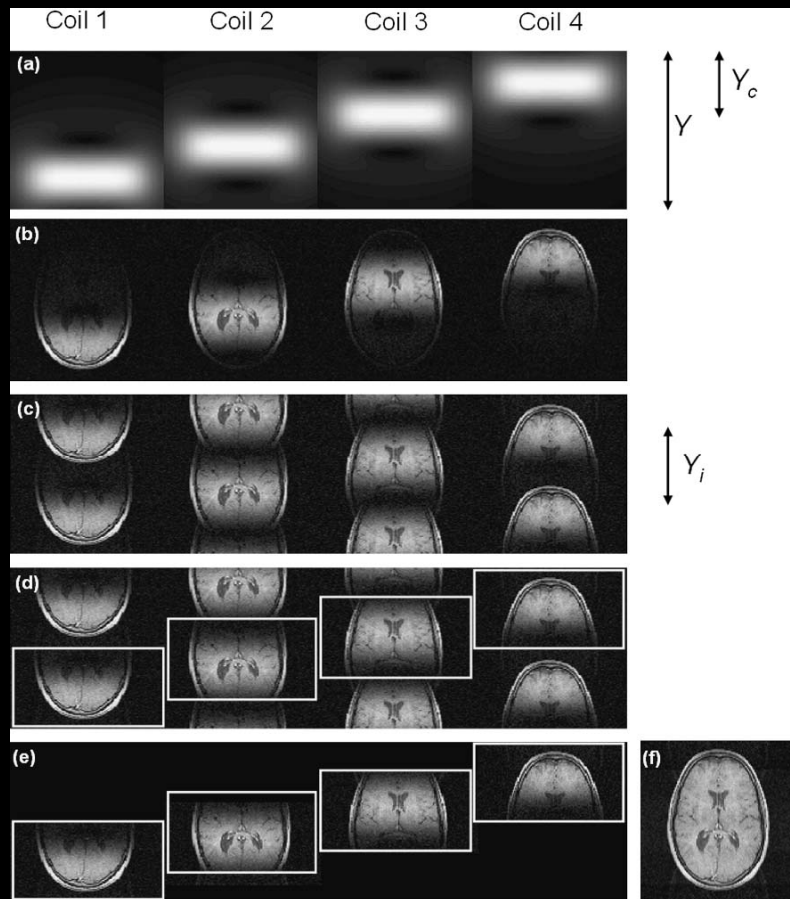
# Parallel Imaging

(see Blaimer et al “ Topics in MRI, 15, 4, 2004” for review article)



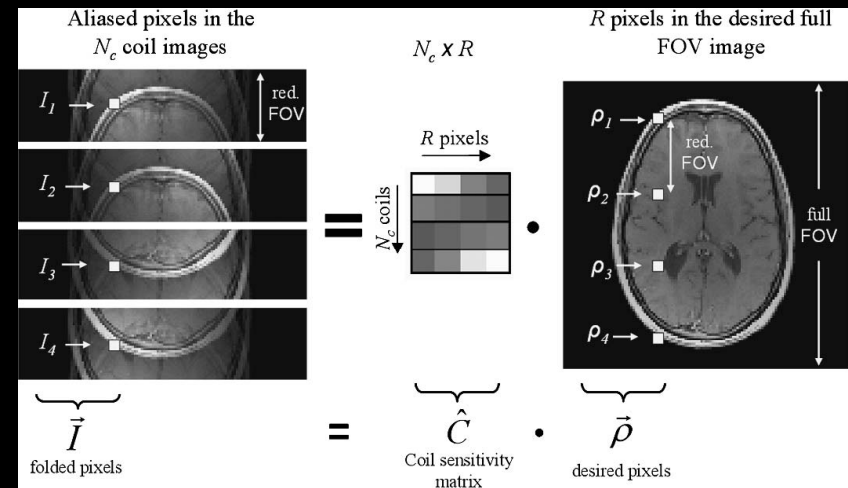
# Parallel Imaging

(see Blaimer et al "Topics in MRI, 15, 4, 2004" for review article)



PILS

## SENSE

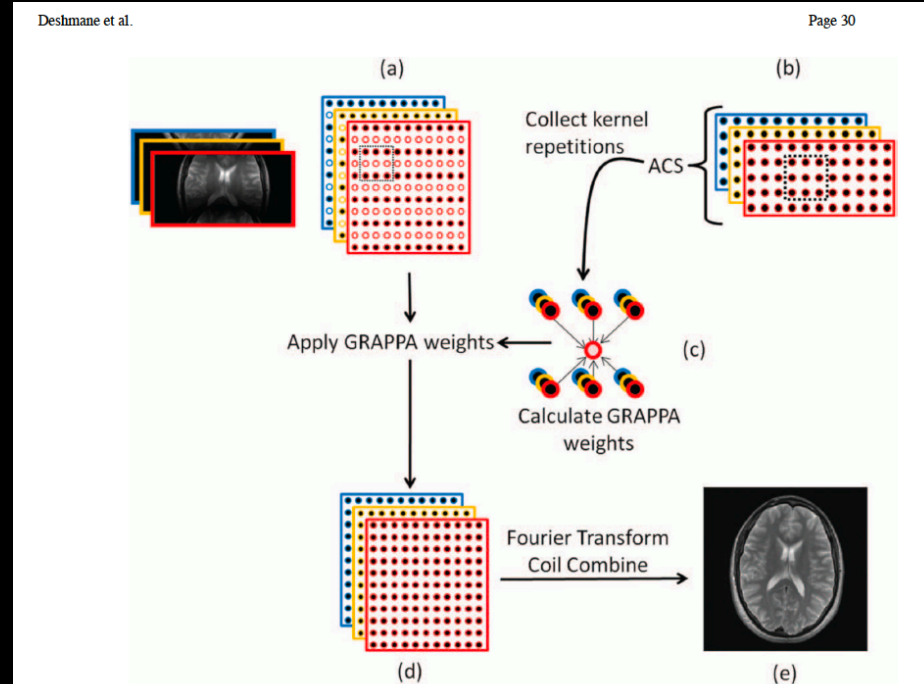


# GeneRalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

Mark A. Griswold,<sup>1\*</sup> Peter M. Jakob,<sup>1</sup> Robin M. Heidemann,<sup>1</sup> Mathias Nittka,<sup>2</sup> Vladimir Jellus,<sup>2</sup> Jianmin Wang,<sup>2</sup> Berthold Kiefer,<sup>2</sup> and Axel Haas

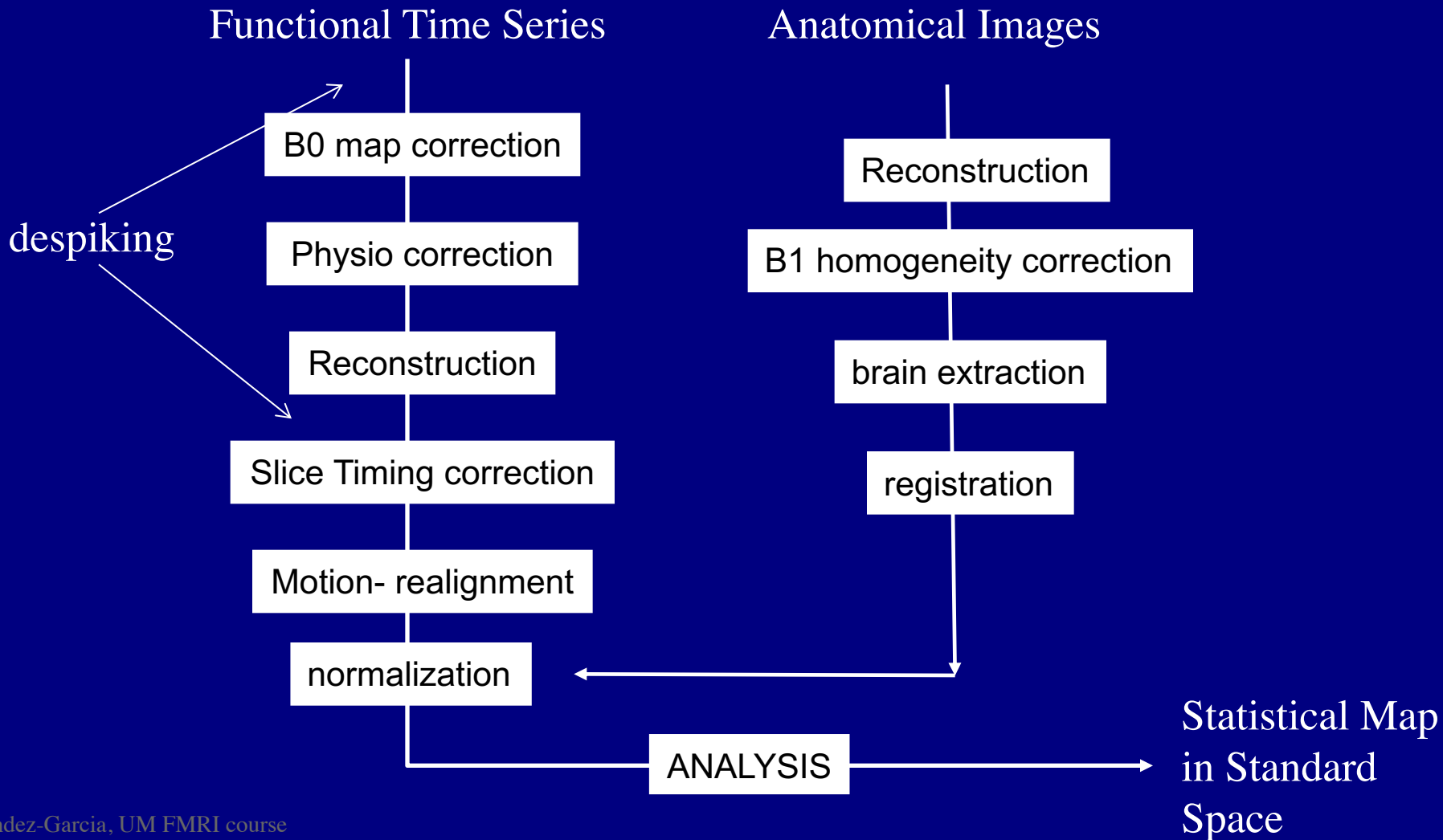
1. Collect undersampled image time series with multiple coils.
2. Collect **some** of the missing data (just once).
3. Calculate interpolation kernel using the multi-coil fully sampled data
4. Interpolate the missing k-space data from the existing data and the interpolation kernel

Deshmane, Anagha & Gulani, Vikas & Griswold, Mark & Seiberlich, Nicole. (2012). Parallel MR imaging. Journal of magnetic resonance imaging : JMRI. 36. 55-72. 10.1002/jmri.23639.

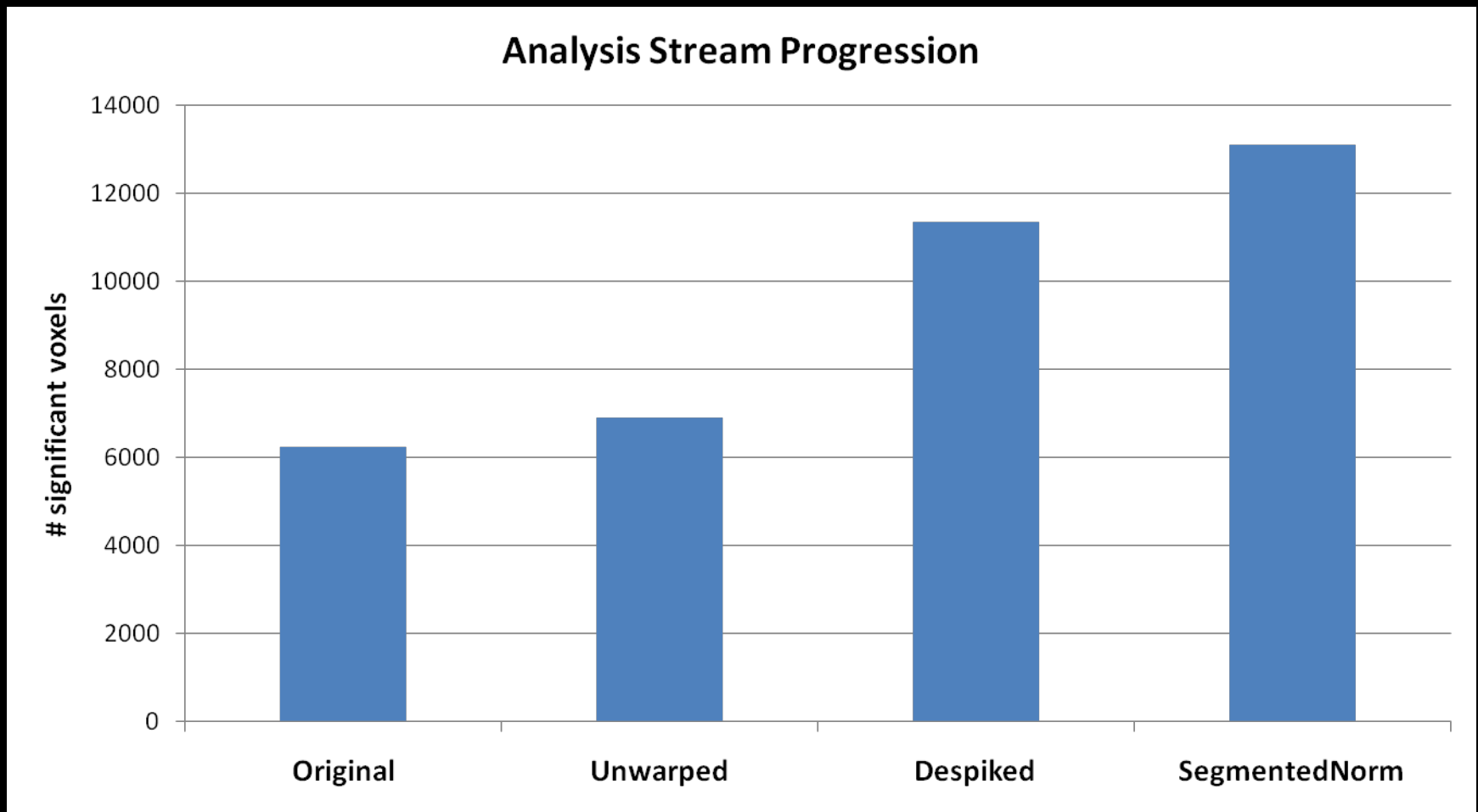


**Figure 9.**  
**a:** Undersampled  $k$ -space data are collected from each coil, where the different coils are shown with different colors. The kernel (outlined by the dotted black box) consisting of some source points (solid circles) and target points (empty circles) defines the neighborhood of  $k$ -space points that will be used for the GRAPPA reconstruction. **b:** Additional data (auto-calibration signals, or ACS) are collected, usually near the center of  $k$ -space. **c:** The repetitions of the kernel through the ACS region are used to calculate the GRAPPA weights. **d:** The GRAPPA weights are then applied to fill in the missing  $k$ -space data from each coil to produce fully sampled single-coil data. **e:** The Fourier transform is used to obtain single-coil images, which are then combined to form a reconstructed full-FOV image. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

# Putting it all together :



# How important is pre-processing?



Appendix ...

# Local interpolation (use a few neighbors at a time)

- Multiplication in frequency = Convolution in time
- Instead of the Frequency domain phase shift, calculate a **weighted average** of a few neighbors neighbors
  - Weights are determined by the **sinc()** function
  - Prevents local errors from affecting the whole time series
  - Can be faster
  - Can build filters onto the sinc function.
- Drawbacks:
  - ringing artifacts if not enough points are used,
  - Too slow if too many points are used in window.

# Does it matter how we interpolate?

- Alternatives to the sinc() function:
  - Nearest Neighbor
  - Linear , Bilinear, Trilinear ...Polynomial
- They can all be shown to be some sort of weighted average : a convolution with a different kernel ... different properties
- They are all approximations based on some assumptions about the function. Sinc is most accurate as long as enough data are used.
- These concepts also apply to image interpolation, resampling, etc.



# “Localized” interpolation: the long version

The digitized signal in frequency domain

$$F(\omega) e^{-i\omega\tau}$$

$$\text{rect}(\omega)$$

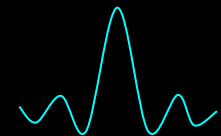
$$\text{rect}(\omega)$$



The signal in time domain

$$f(t) * \text{sinc}(t - \tau)$$

$$\text{sinc}(t) = \sin(t) / t$$



Convolution:

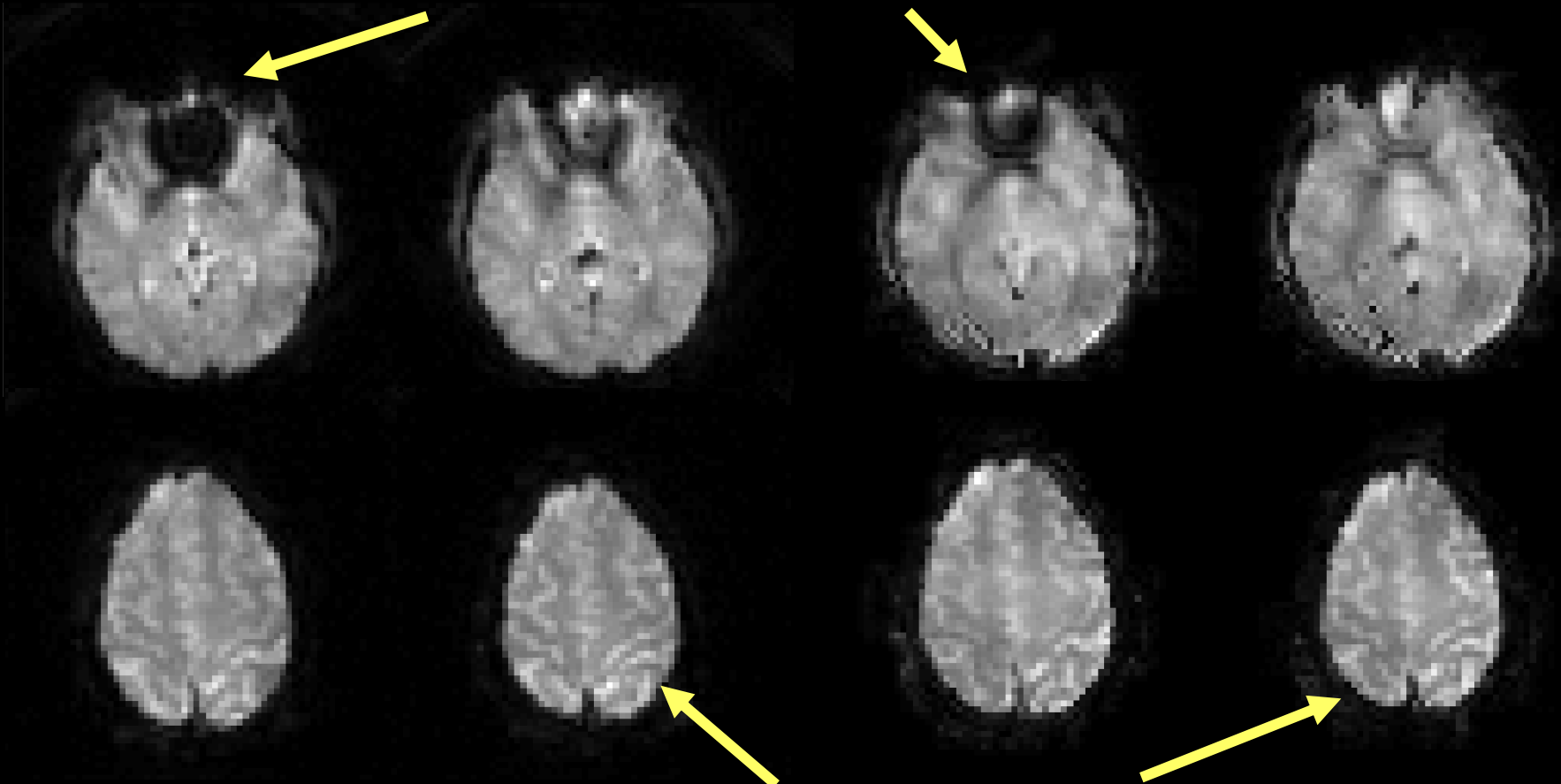
$$f[n] * g[n] = \sum_k f[k] g[n-k]$$

# Spiral SENSE – Results

Head Coil

4-Channel SENSE Coil

**Reduced Susceptibility Artifact**



**Excellent Detail**

Hernandez-Garcia, UM FMRI course