MRI Physics:

Magnetic Field Gradients

Nicole Seiberlich
Associate Professor, Radiology
Co-Director of MIITT

Pulse Sequences

- Contrast (Spin Preparation)

  What kind of contrast does the image have?
  What is the TR, TE, Flip Angle, etc.?
  Gradient echo/spin echo/etc.

- Localization (Image Acquisition)

  How is the image acquired?
  How is “k-space” sampled?
  Spatial Resolution? Field-of-View?

Goals of Image Acquisition

- Acquire 2D (or 3D) Fourier data
- Acquire samples finely enough to prevent aliasing (FOV)
- Acquire enough samples for the desired spatial resolution (Δx)
- Acquire images with the right contrast
- Do it fast as possible
- Do it without distortions and other artifacts

Magnetic Fields in MRI

B_0 – The main magnetic field

- Always on (0.5-7 T)
- Magnetizes the object to be imaged
- After excitation, the magnetization precesses around B_0 at $\omega_0 = \gamma B_0$

B_1 – The rotating RF magnetic field

- Tips magnetization into transverse plane
- Used for “excitation”
- On for brief periods, then off

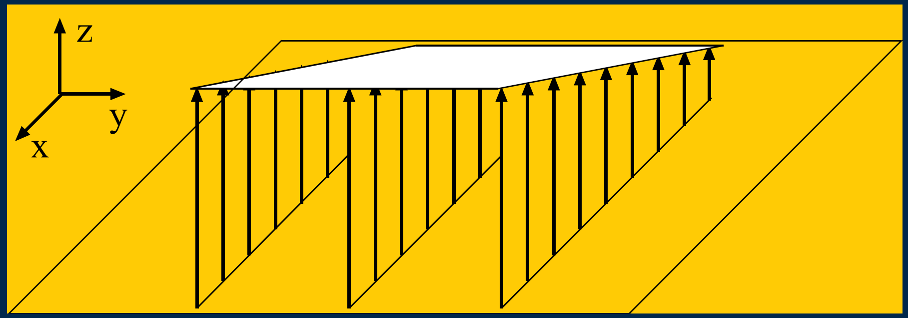
Magnetic Fields in MRI

The last magnetic field to be used in MRI are the gradient fields

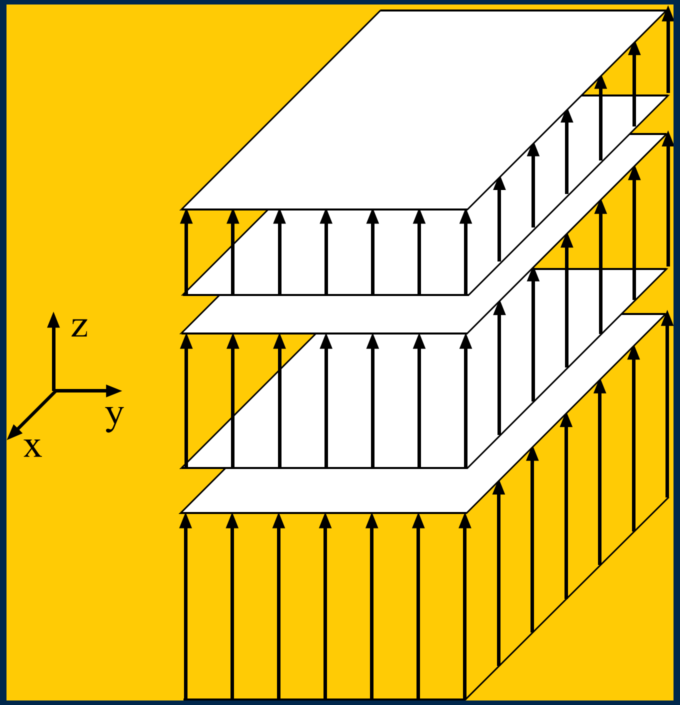
- 3 of them: G_x, G_y, G_z
- Used primarily for spatial localization
- Make the magnetic field different in different parts of the body, e.g. for the x-gradient:

$$B(x) = B_0 + G_x \cdot x$$

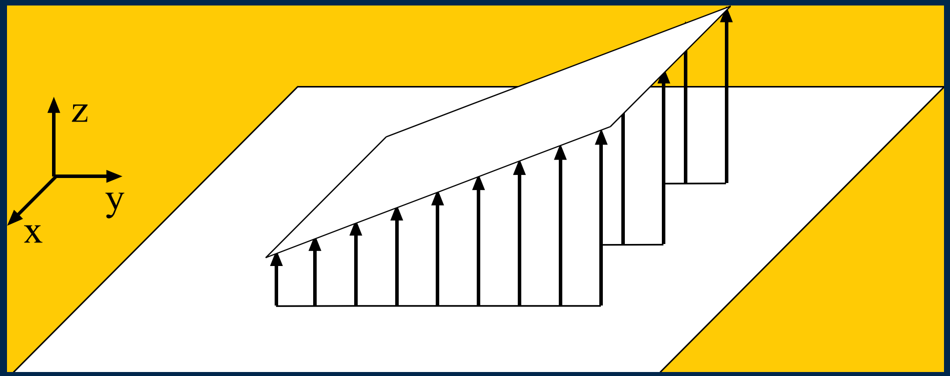
Magnetic Field Gradients



x-gradient (G_x)

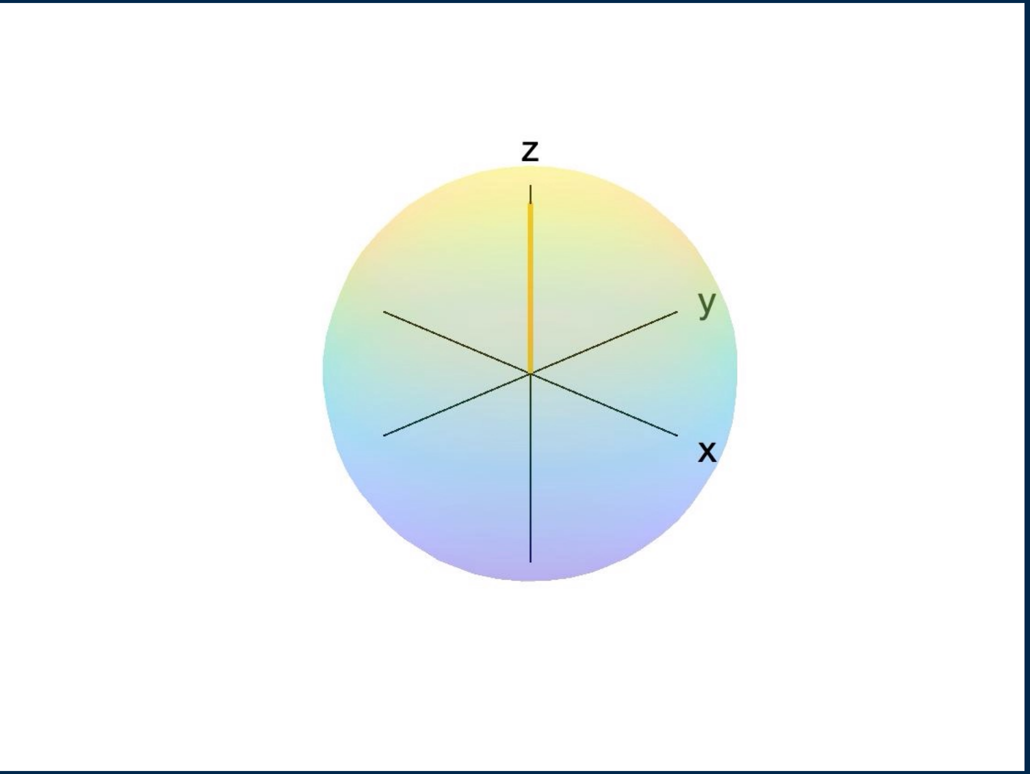


z-gradient (G_z)



y-gradient (G_y)

Precession



$$\omega = \gamma \cdot B_0$$

Larmor Frequency
Angular Frequency

Main Magnetic Field

Gyromagnetic Ratio

B_0=1.5T → ω = 64 MHz → 64 million rounds/sec

B_0=3.0T → ω = 128 MHz → 128 million rounds/sec

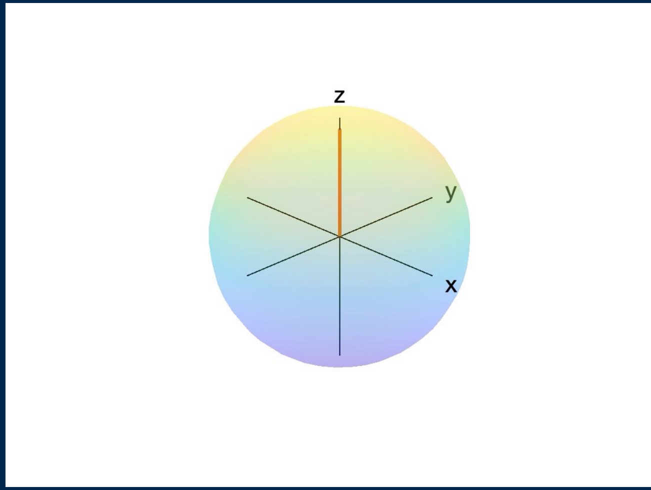
Stronger Magnetic Field = Faster precession

Precession Frequency can be altered with magnetic field gradients

$$B(x) = B_0 + \mathrm{G_x\,x}$$

$$\omega = \gamma \cdot B$$

$$\omega(x) = \gamma B(x) = \gamma(B_0 + \mathrm{G_x\,x})$$

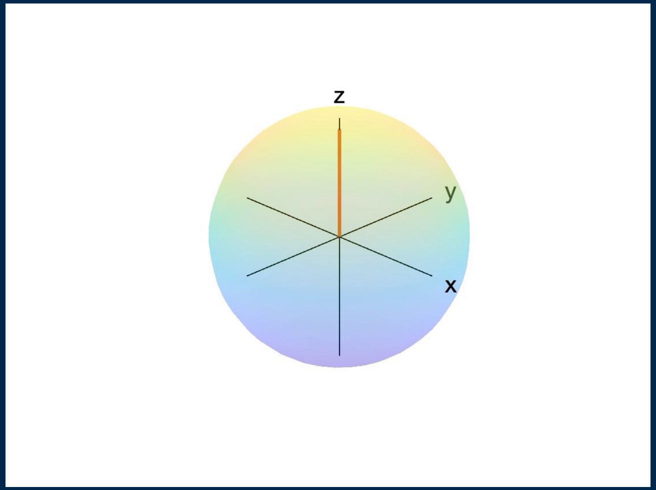


Precession Frequency can be altered with magnetic field gradients

$$B(x) = B_0 + G_x \cdot x$$

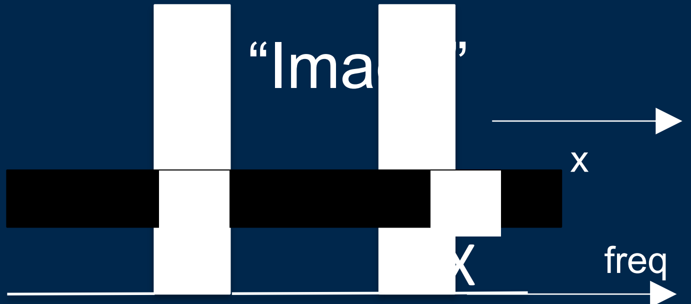
$$\omega(x) = \gamma B(x) = \gamma(B_0 + G_x \cdot x)$$

x=0   x=X



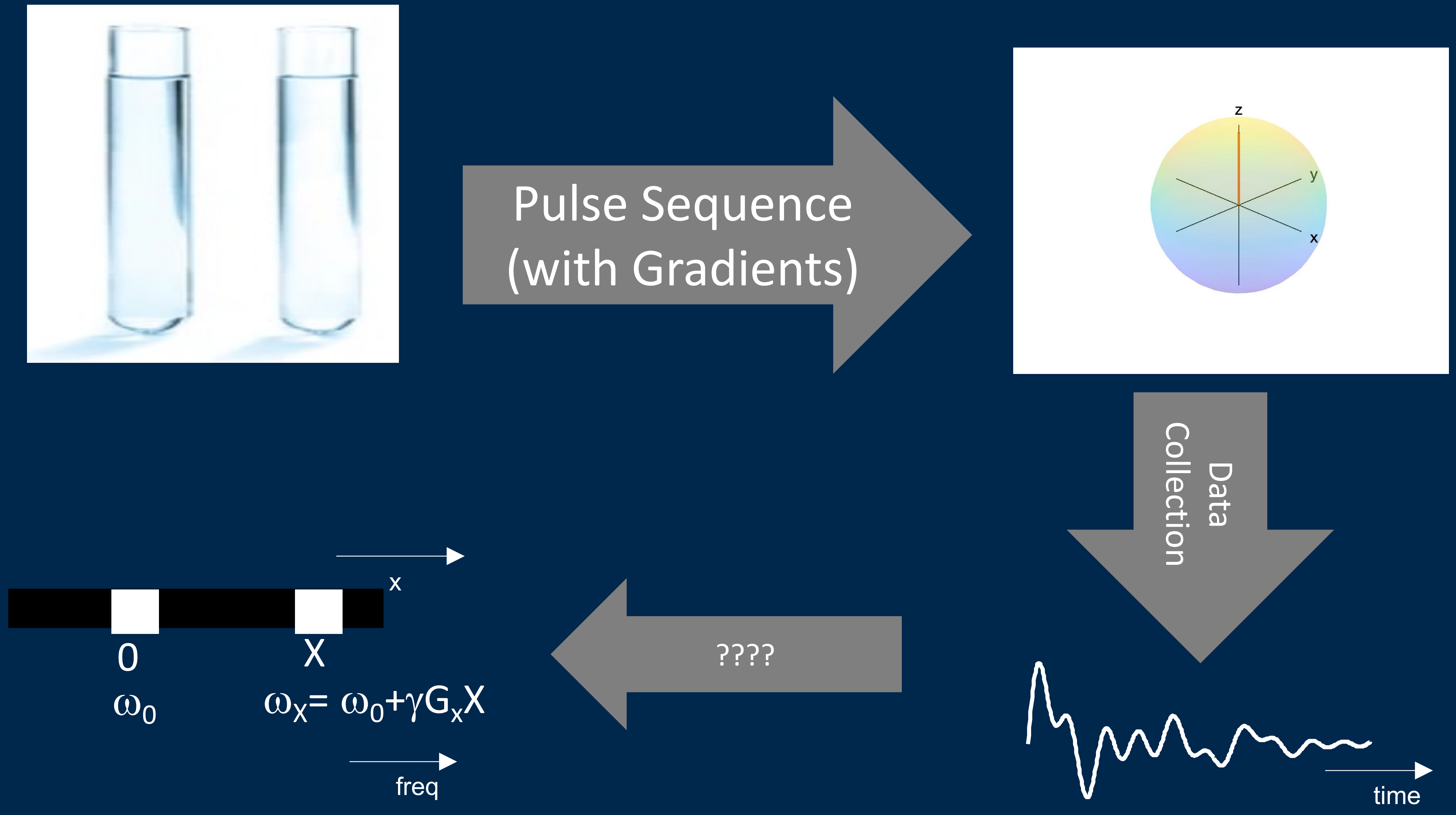
$B=B_0$ $B_X=B_0+G_xX$

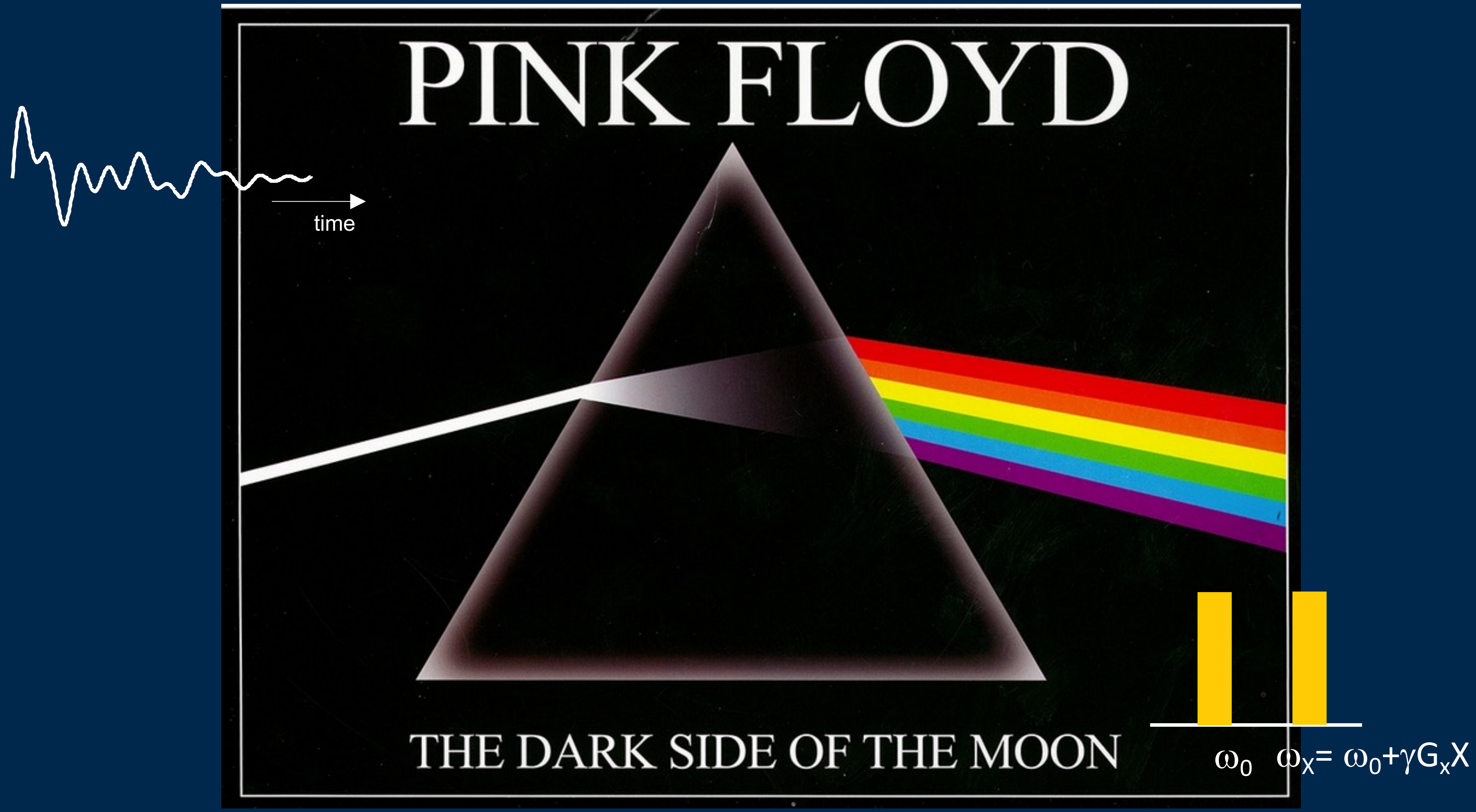
$\omega= \omega_0$ $\omega_X= \omega_0+\gamma G_xX$



ω_0 $\omega_X= \omega_0+\gamma G_xX$

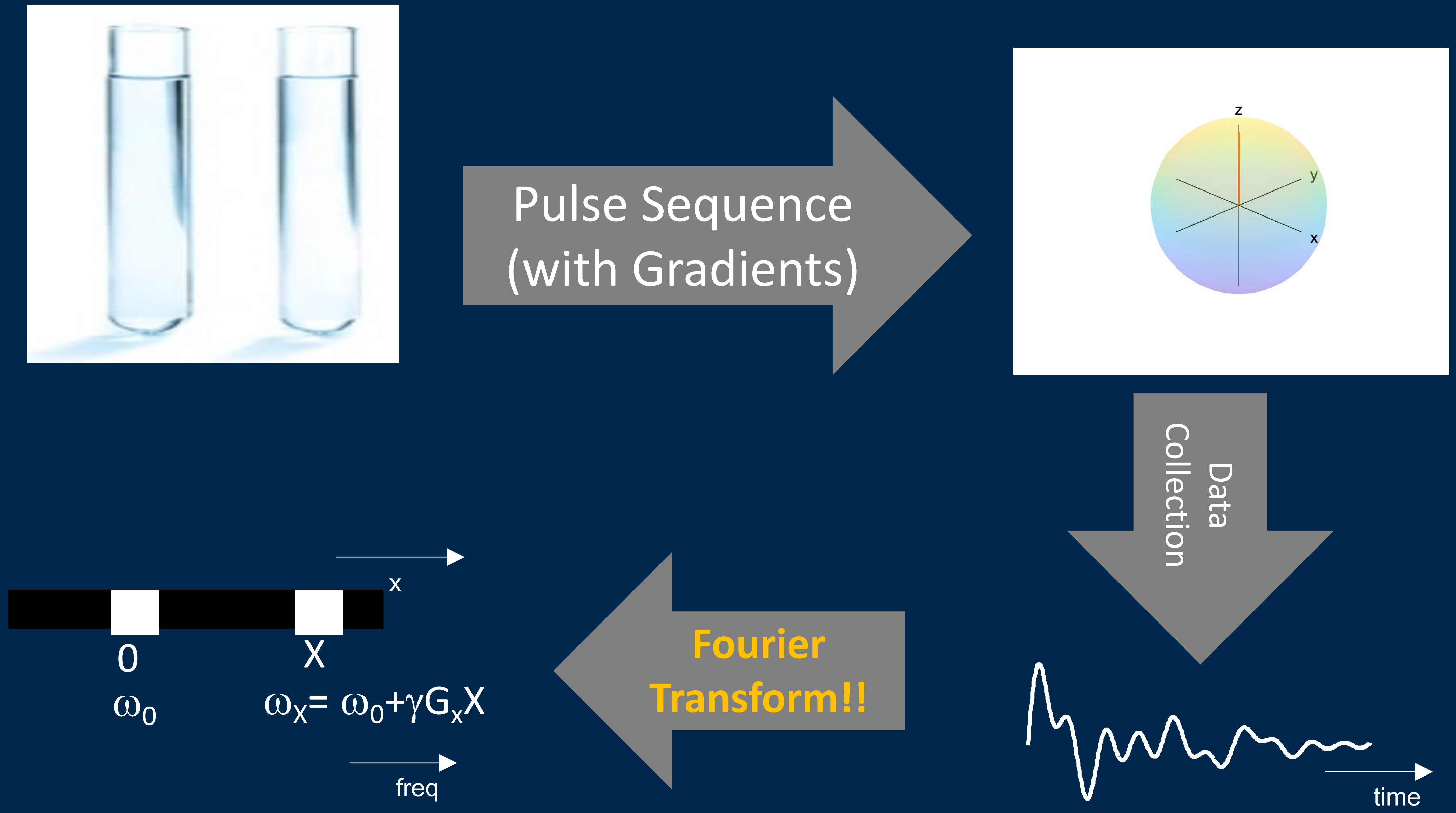
The Challenge



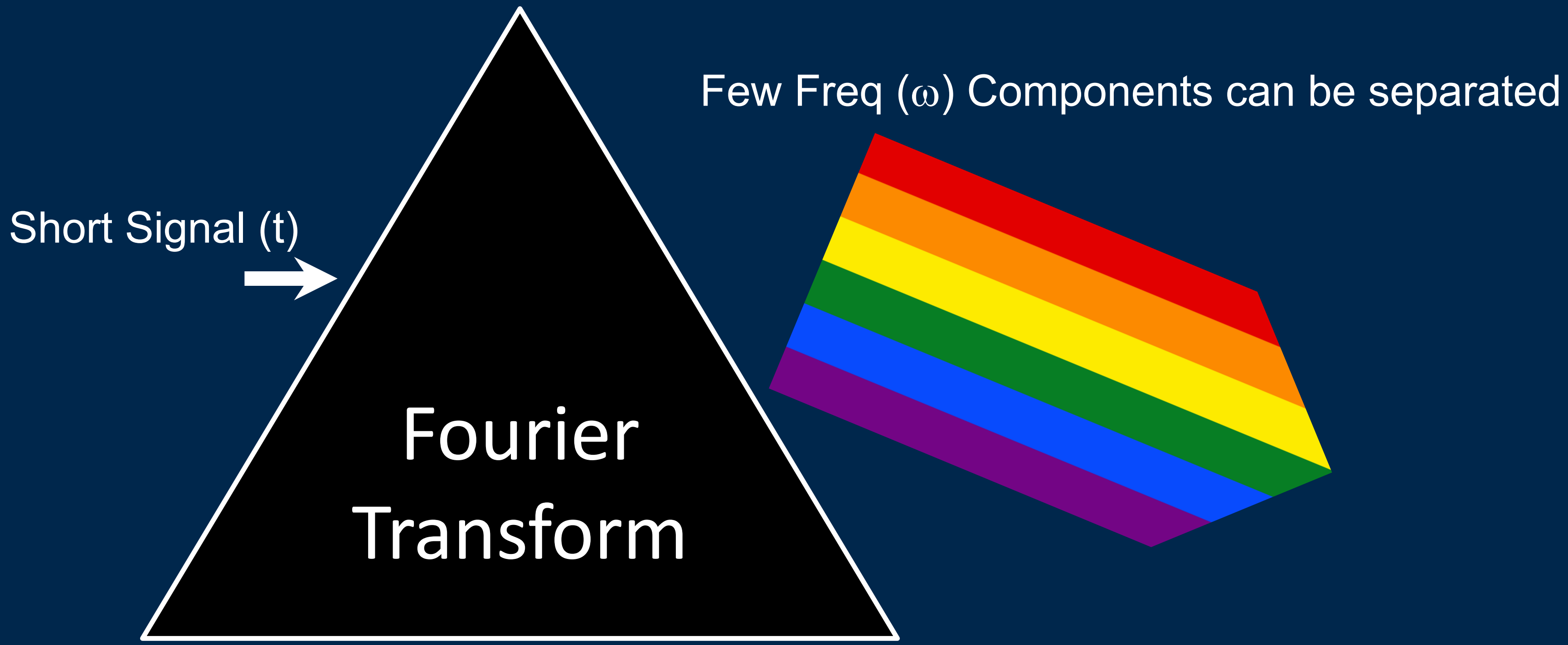


time
PINK FLOYD
THE DARK SIDE OF THE MOON
ω_0
$\omega_X = \omega_0 + \gamma G_x X$

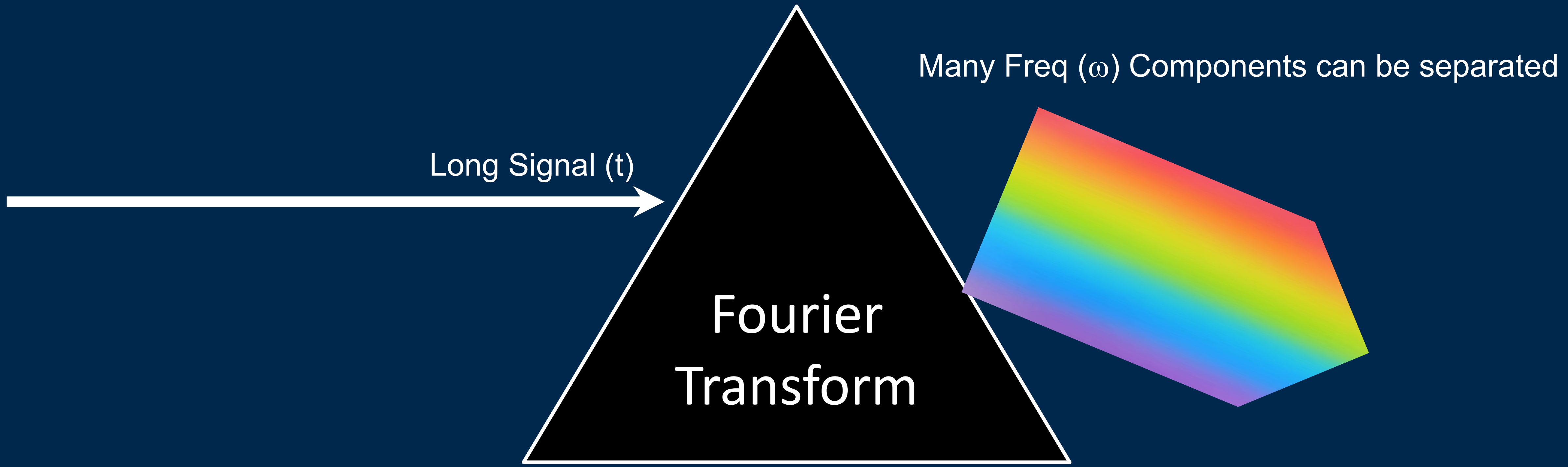
The Challenge



Signal Length → Freq Resolution



Signal Length → Freq Resolution

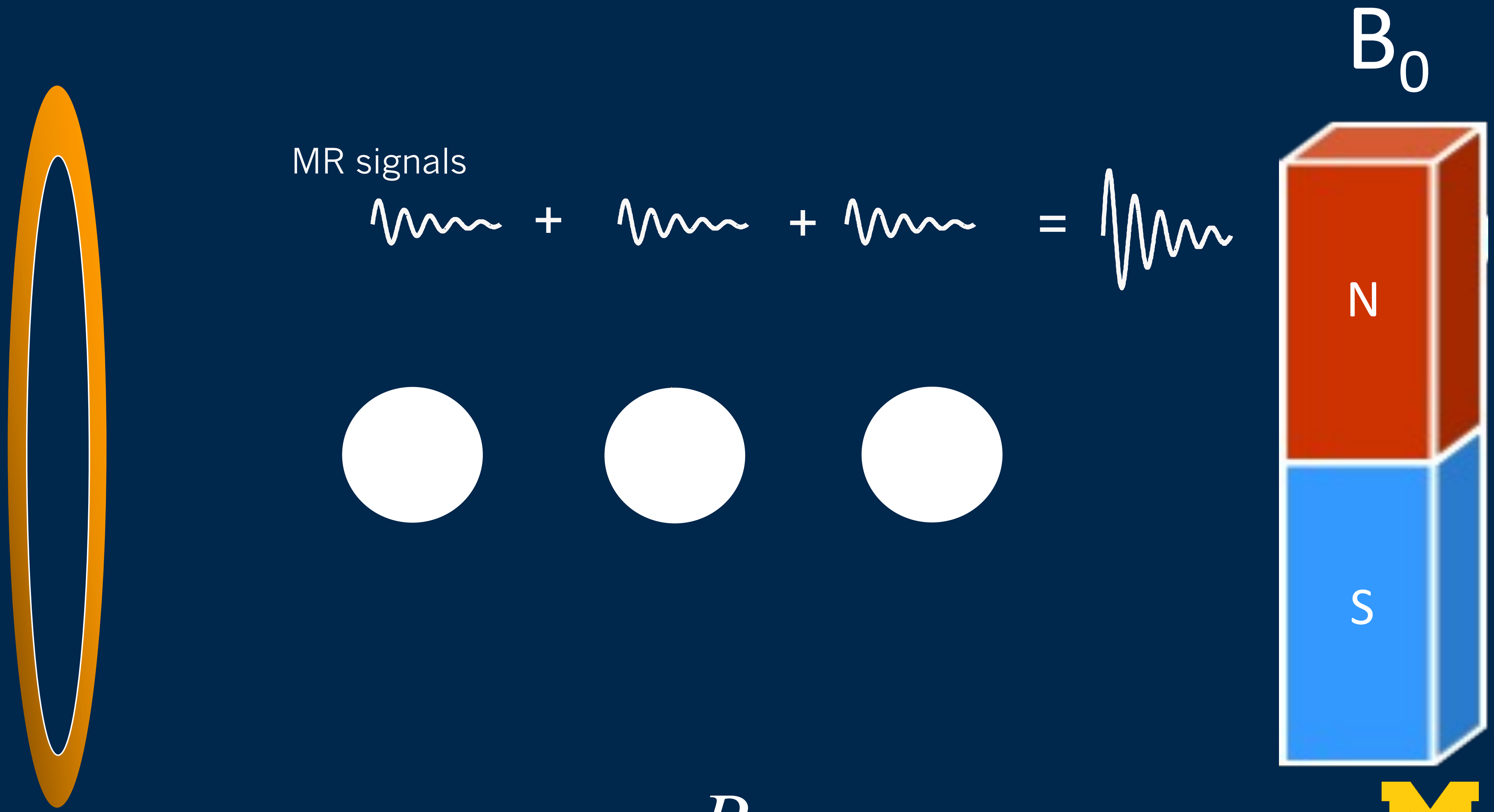


MRI Physics:

Frequency and Phase Encoding

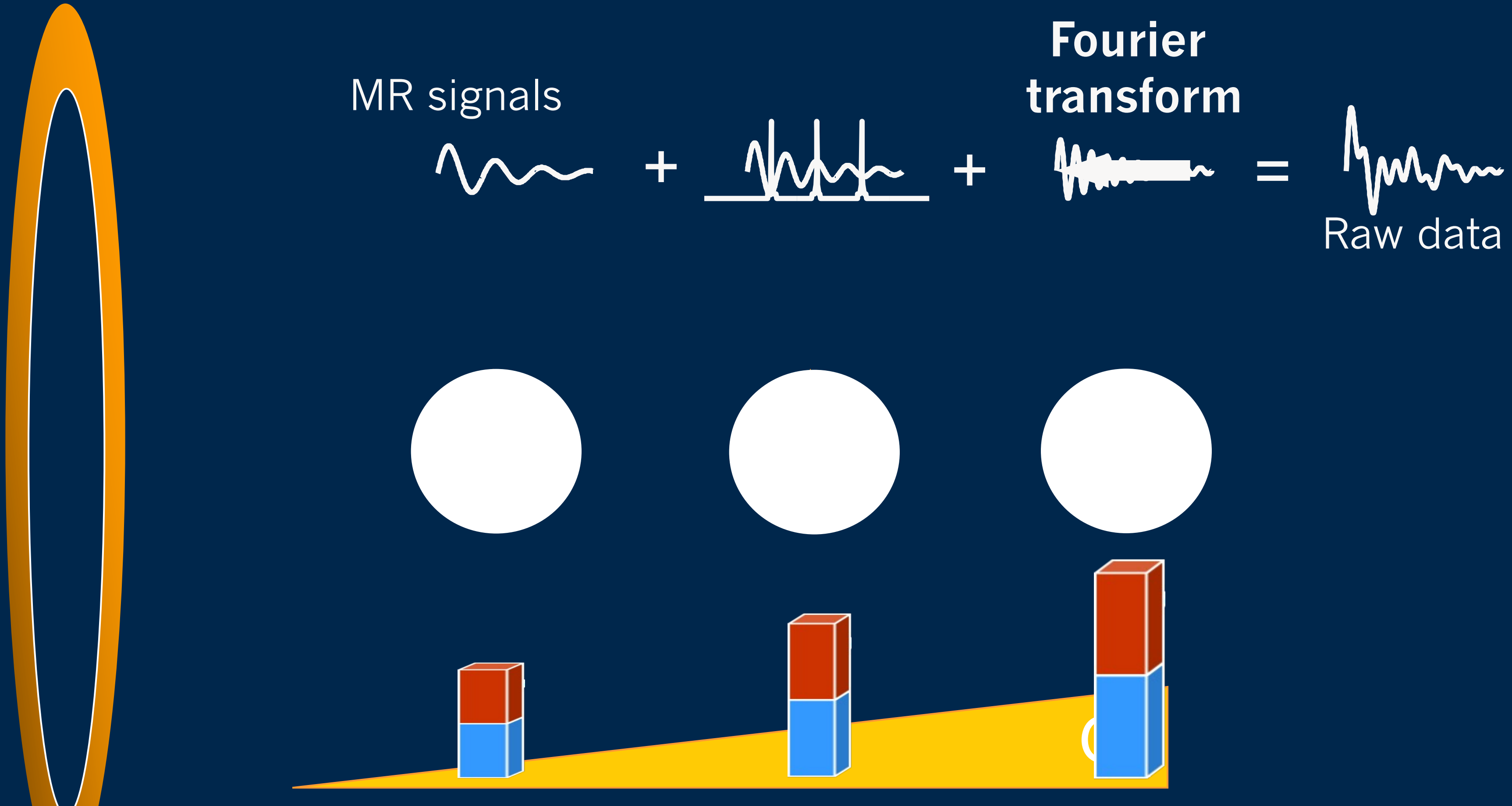
Nicole Seiberlich
Associate Professor, Radiology
Co-Director of MIITT

Spatial Localization



$$\omega = \gamma B_0$$

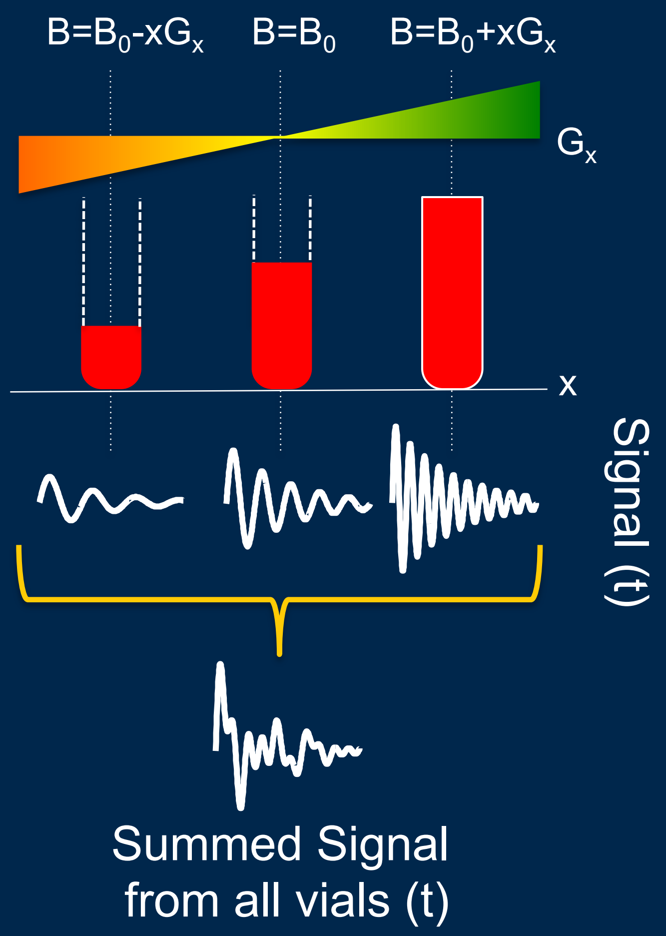
Spatial Localization

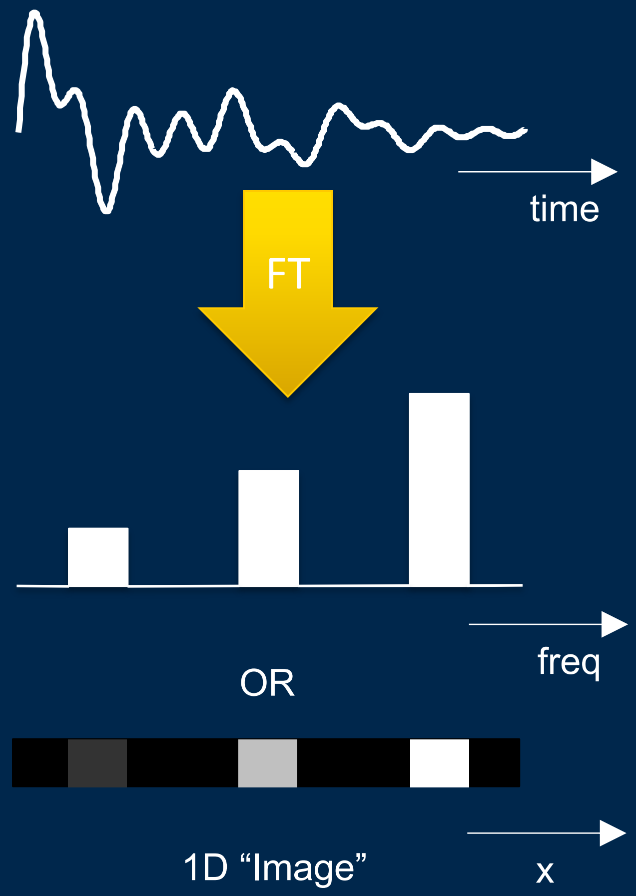


$$\omega(x) = \gamma B(x) = \gamma(B_0 + G_x \cdot x)$$

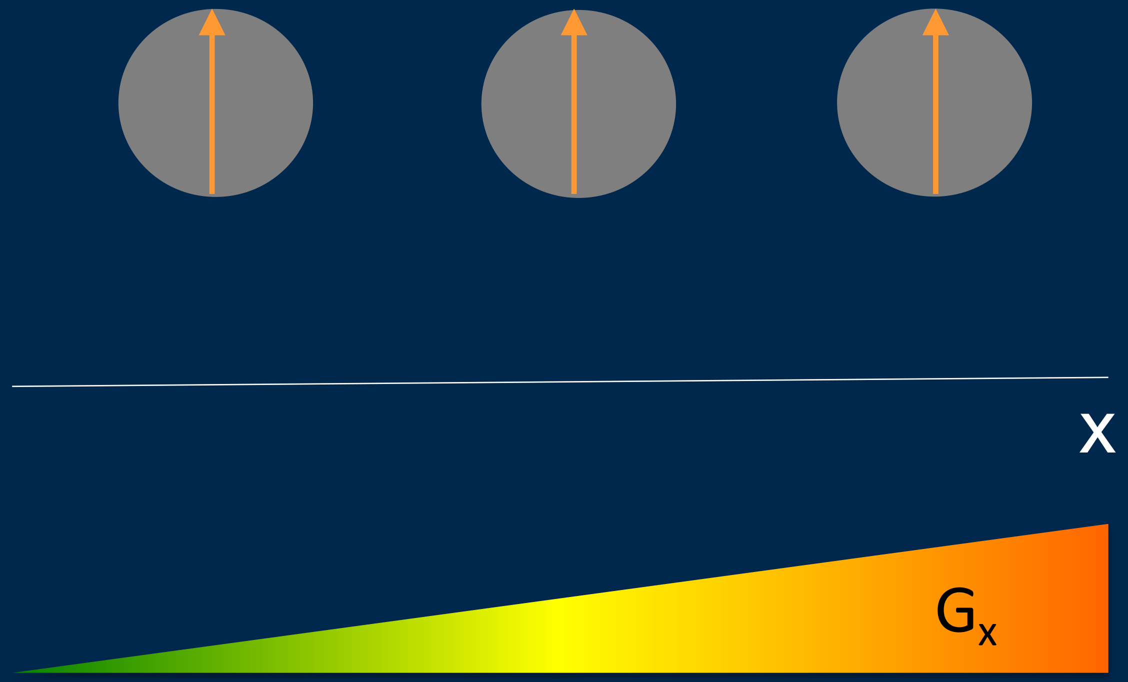
MICHIGAN MEDICINE

Frequency Encoding





Gradients alter frequency and phase of spins

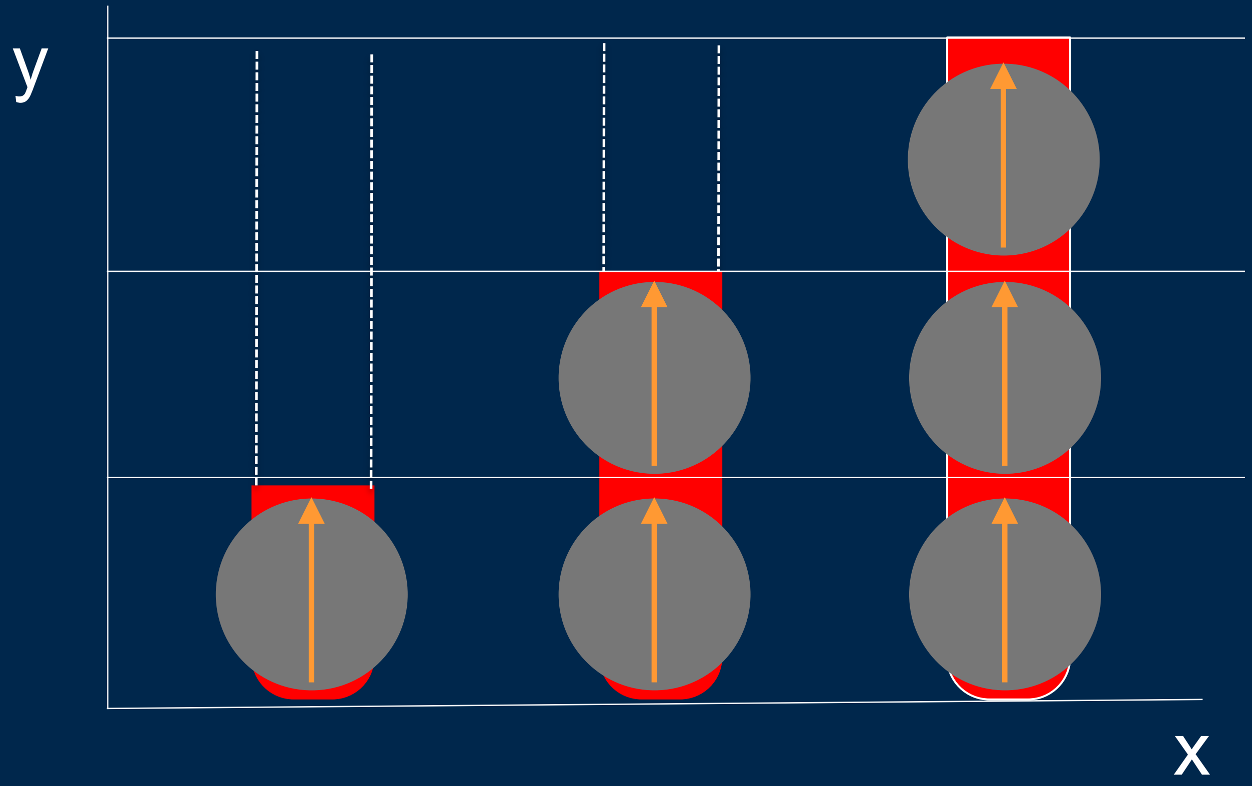


Frequency→how fast spins precess

Phase→relative orientation with respect to one another

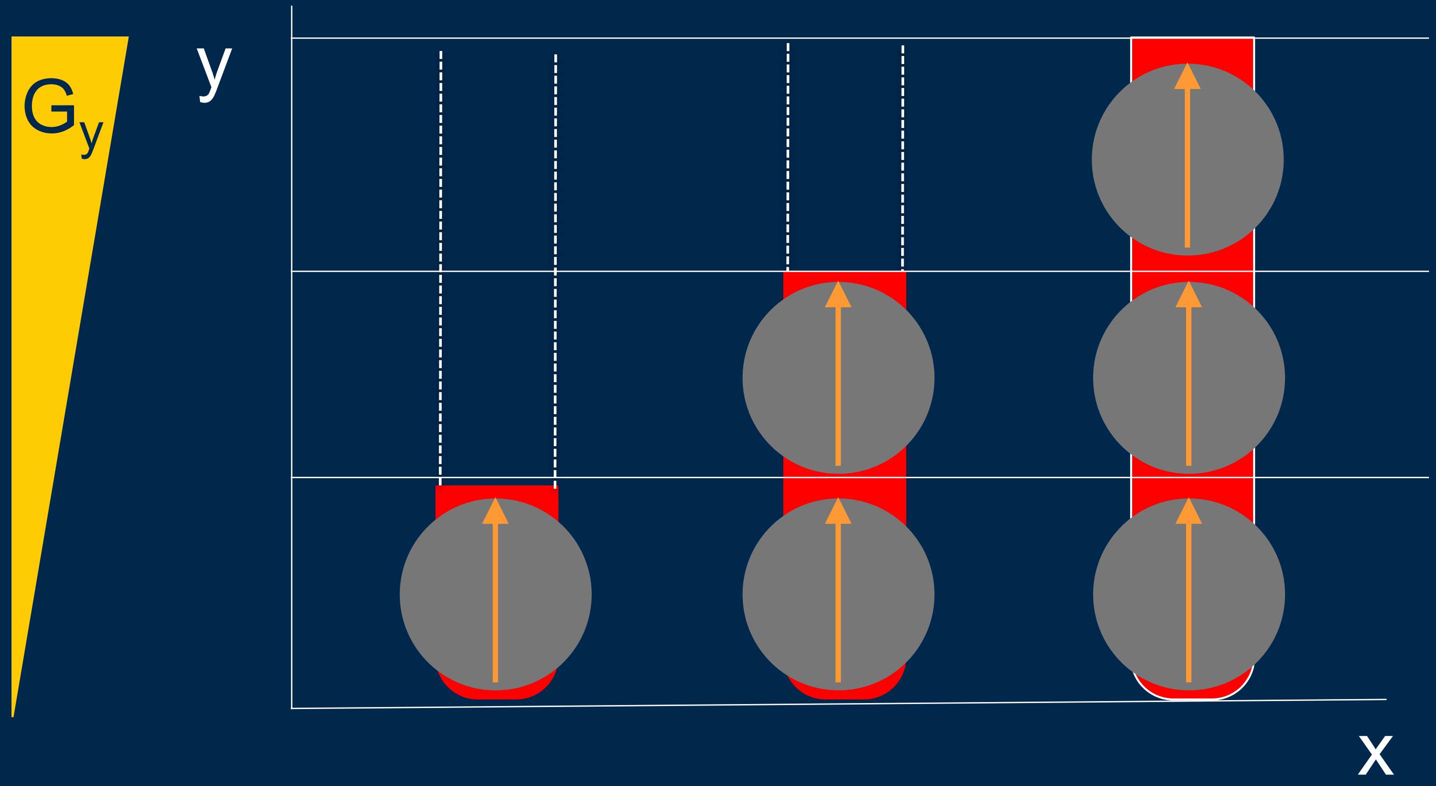
MICHIGAN MEDICINE

Phase Encoding



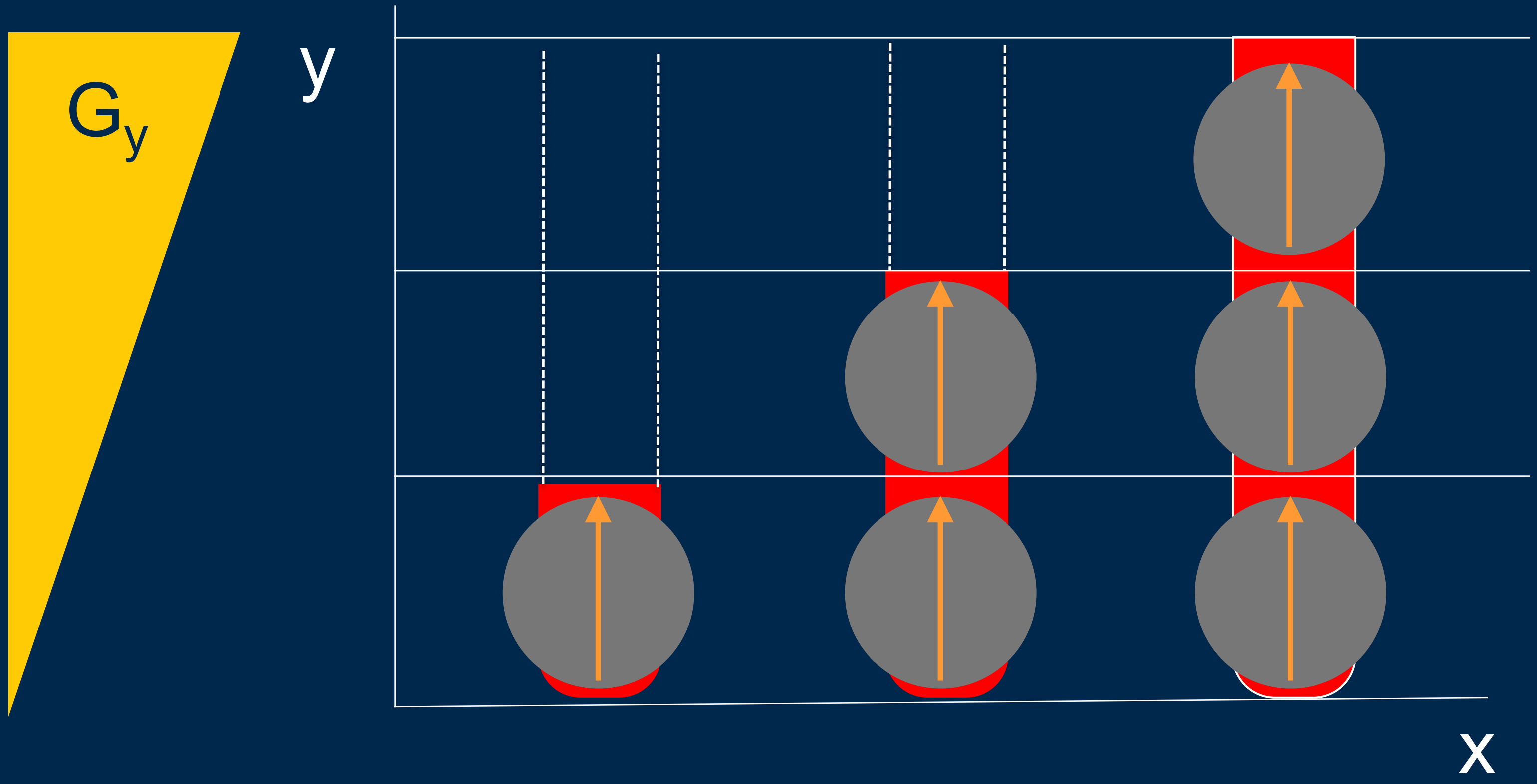
MICHIGAN MEDICINE

Phase Encoding



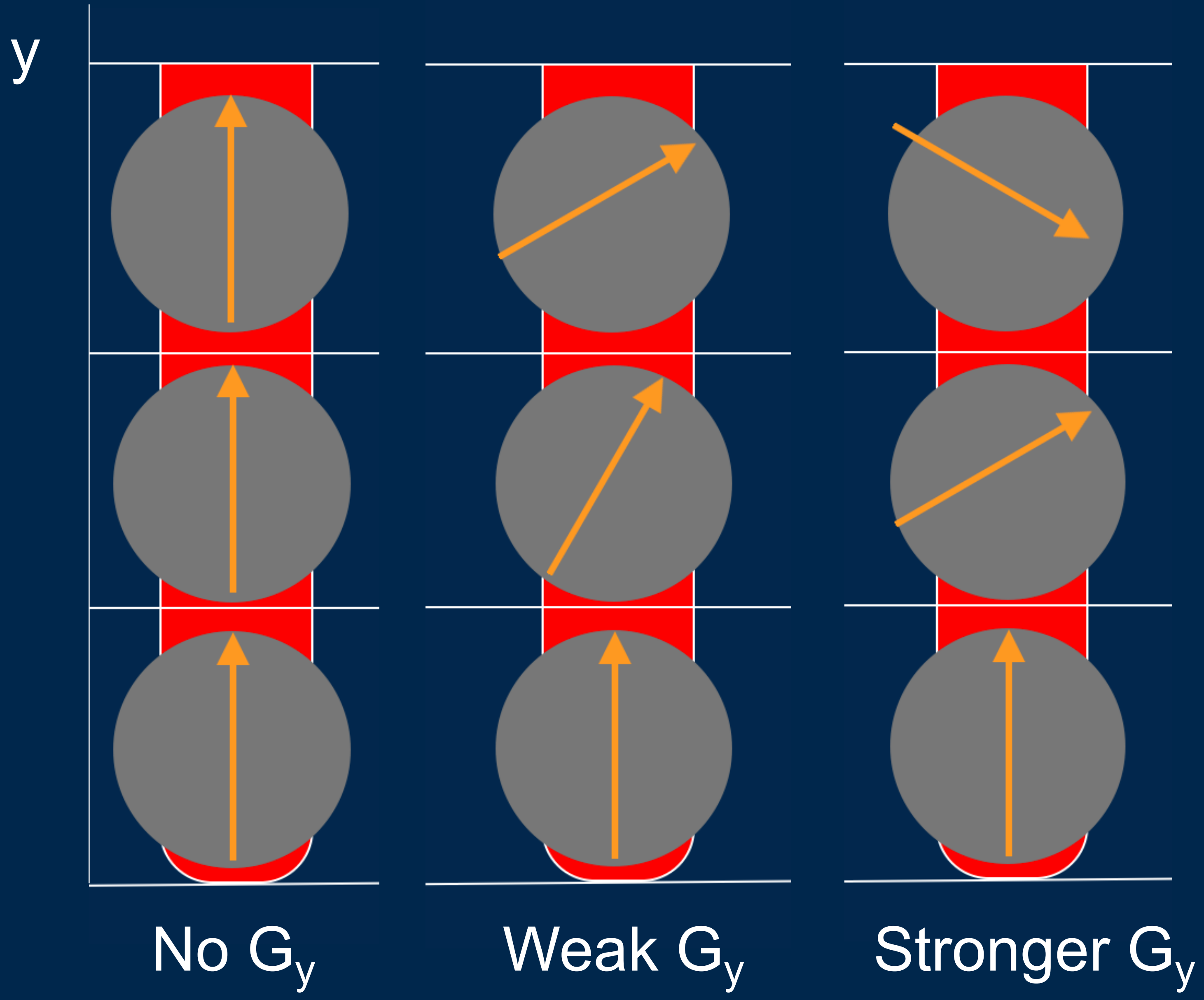
MICHIGAN MEDICINE

Phase Encoding



MICHIGAN MEDICINE

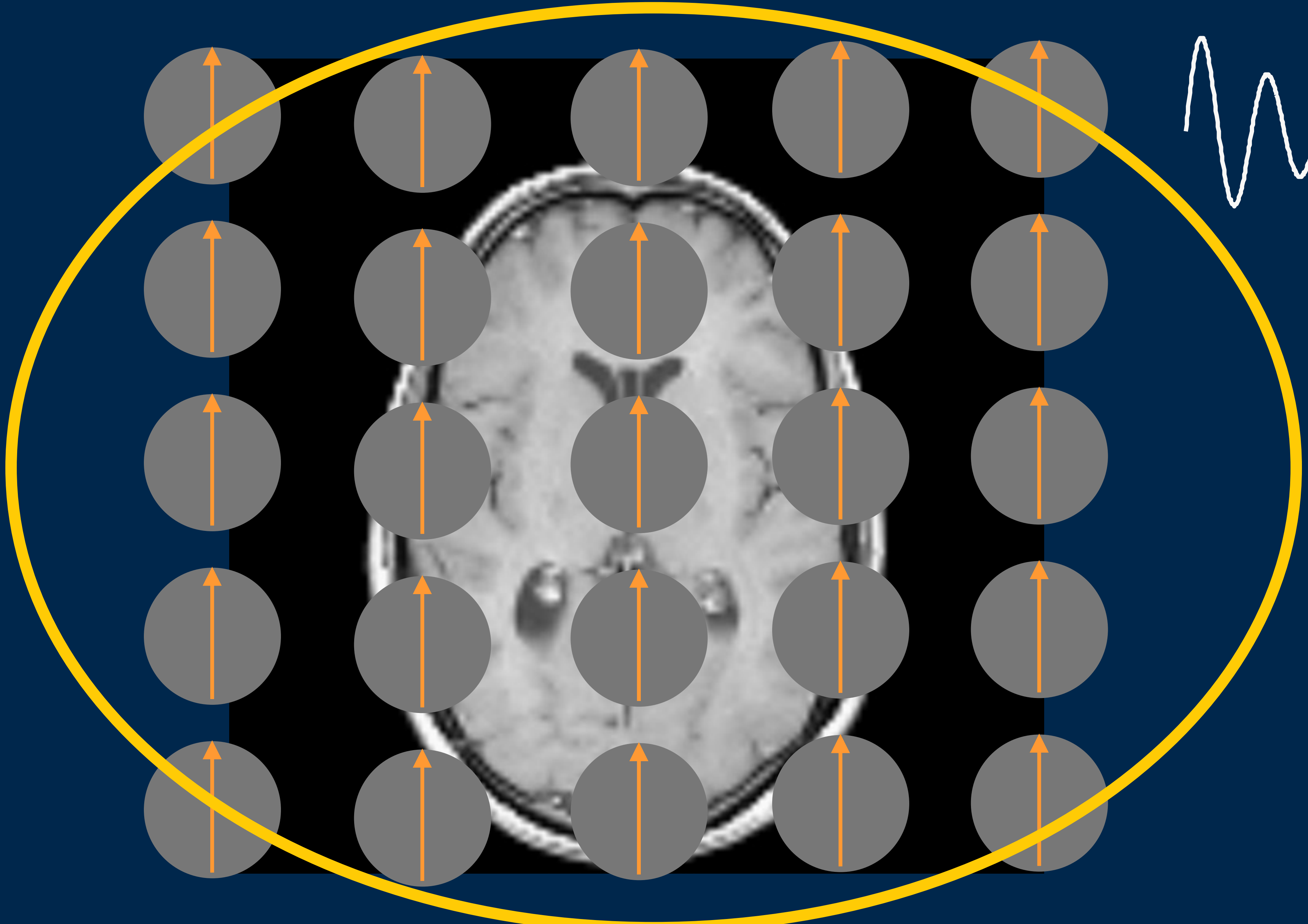
Phase Encoding→Just like Freq Encoding

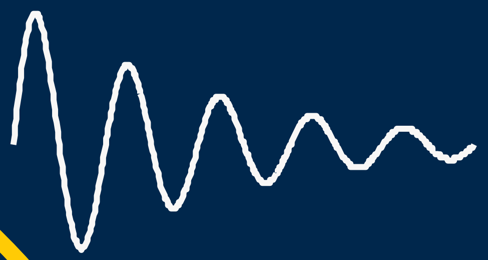


Phase Encoding works just like Frequency Encoding

→Imparts spatial dependent frequency which can be extracted using Fourier Transform

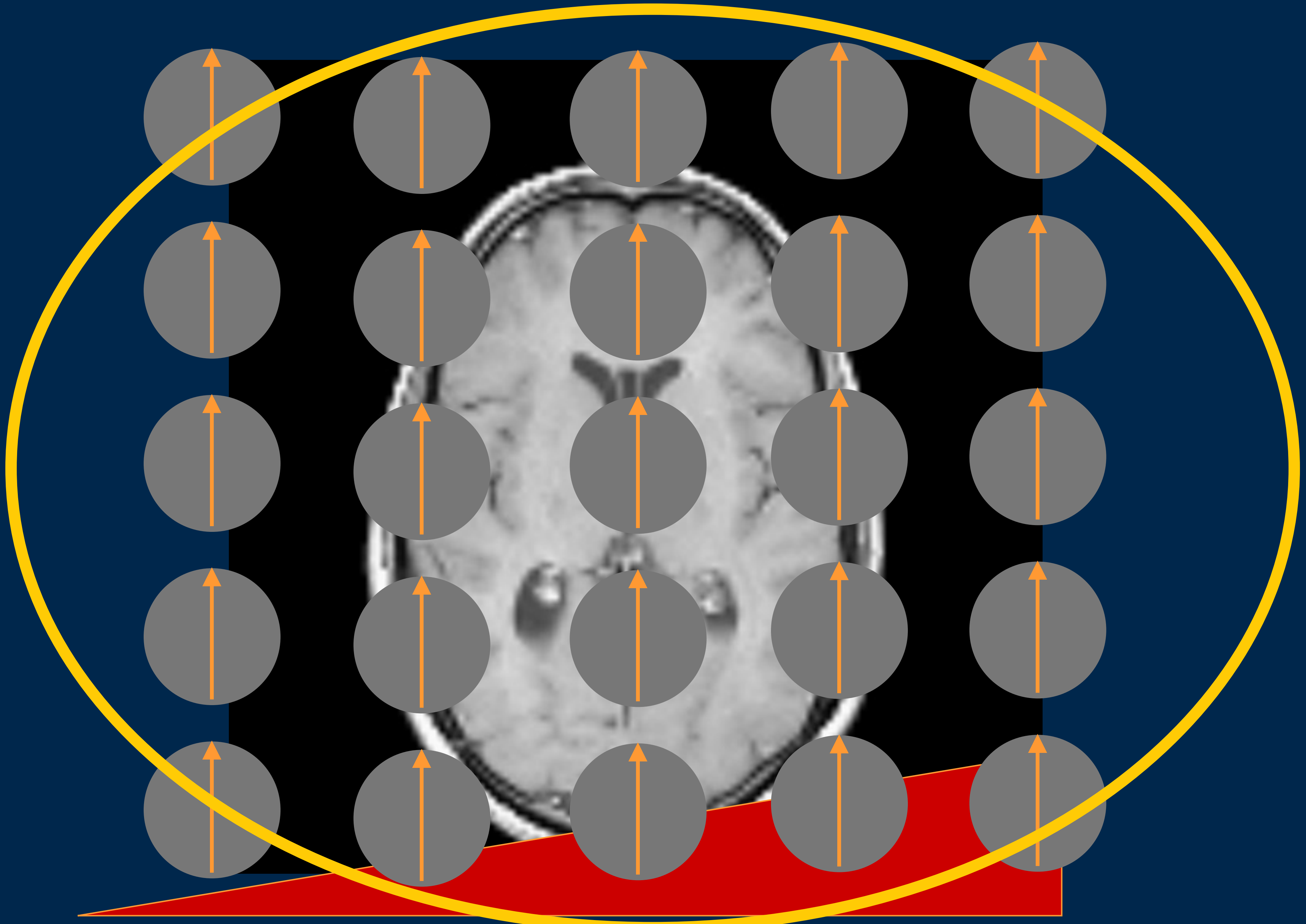
Spins in Constant Magnetic Field



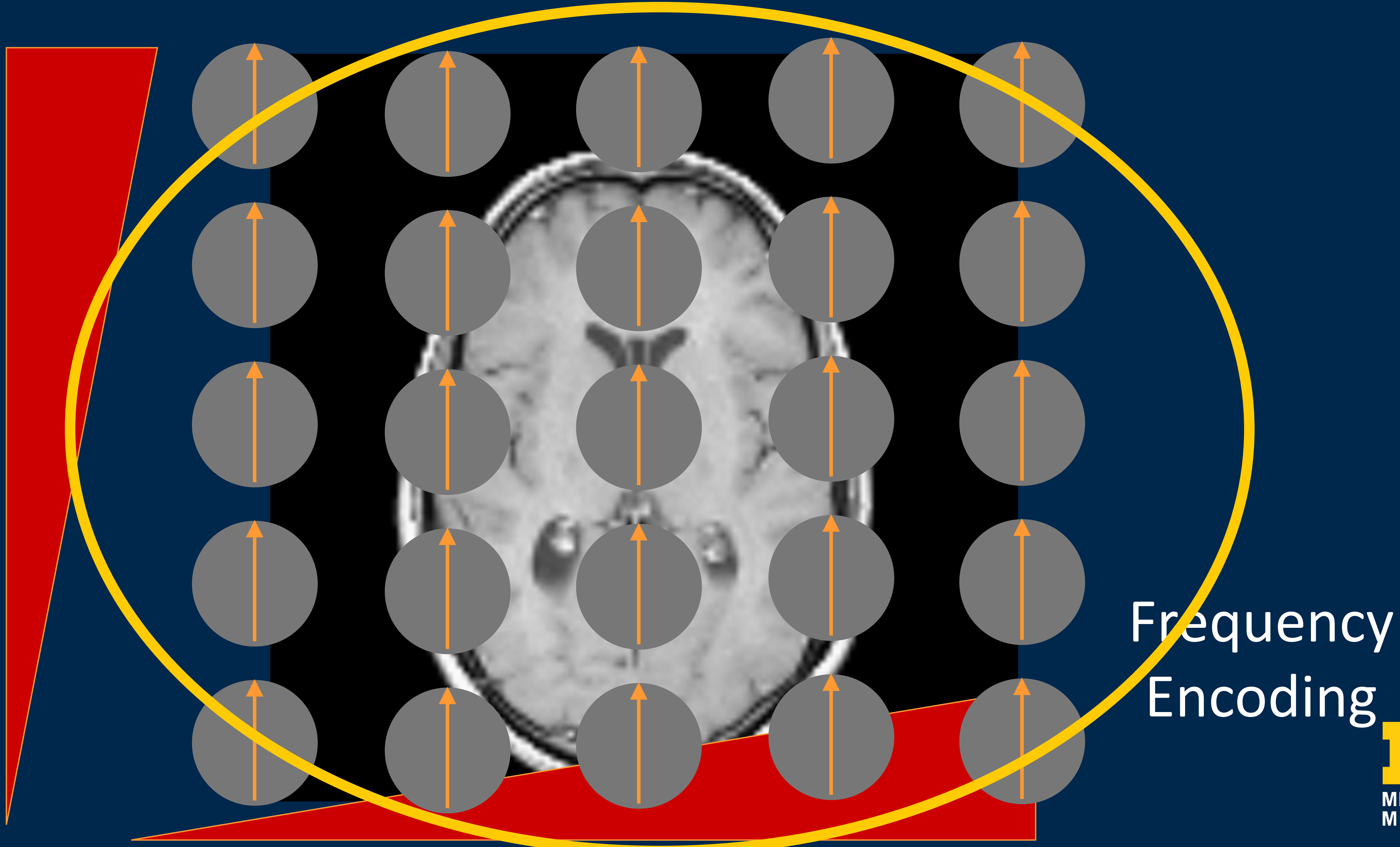


MICHIGAN
MEDICINE

Frequency Encoding



Phase Encoding



Signal Equation and Fourier MRI

$$s(x,y,t) = \rho(x,y) \cdot e^{-i\omega t} = \rho(x,y) \cdot e^{-i\gamma B_0 t}$$

$$s(x,y,t,\tau) = \rho(x,y) \cdot e^{-i\gamma (B_0 + G_x) x \cdot t} \cdot e^{-i\gamma (B_0 + G_y) y \cdot \tau}$$

$$S(t,\tau) = \int\int s(x,y,t,\tau)\, dx\, dy$$

$$= \int\int \rho(x,y) \cdot e^{-i\gamma (B_0 + G_x) x \cdot t} \cdot e^{-i\gamma (B_0 + G_y) y \cdot \tau}\, dx\, dy$$

$$k_x = \gamma (B_0 + G_x) t$$

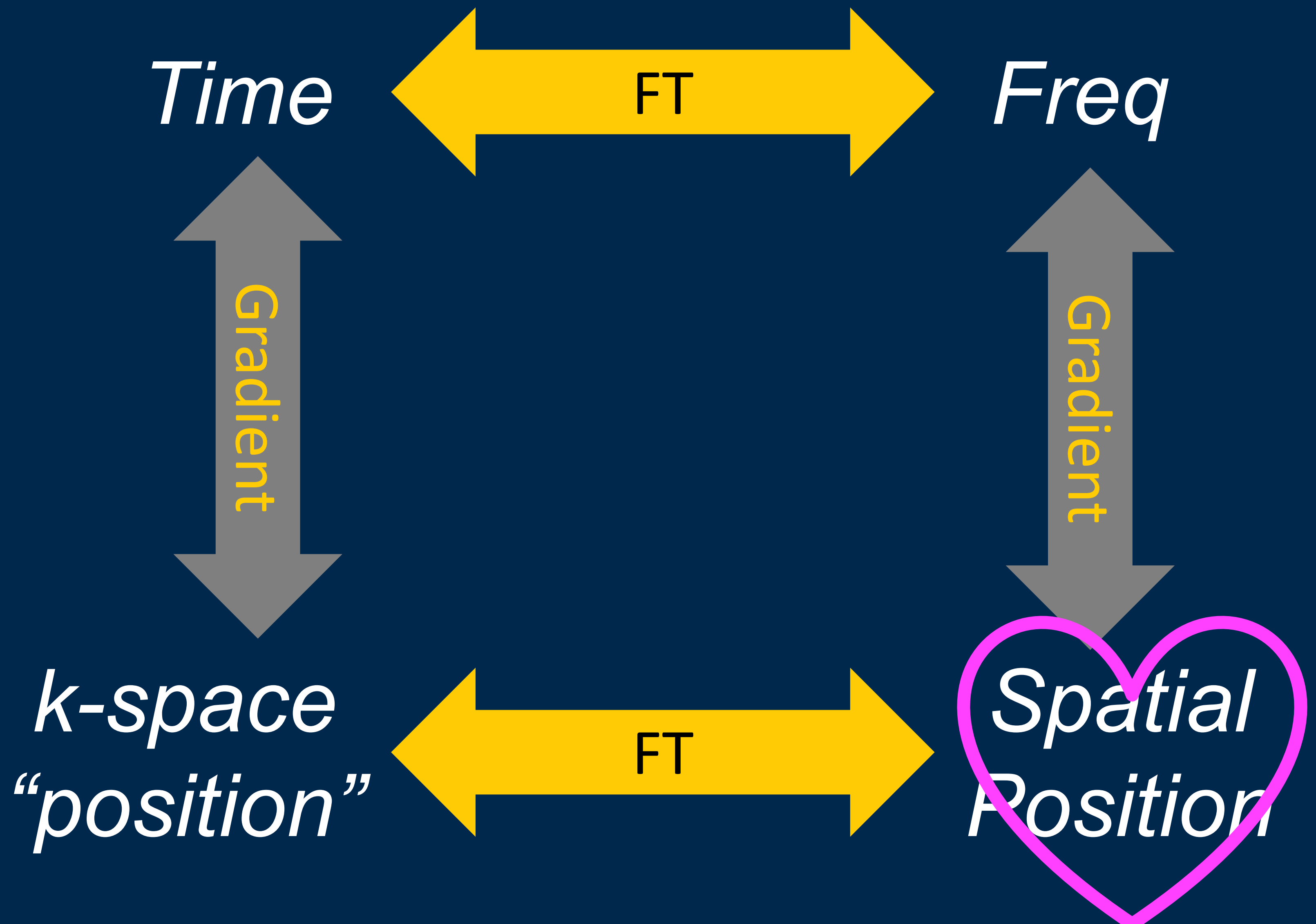
$$k_y = \gamma (B_0 + G_y) \tau$$

$$S(k_x, k_y) = \int\int \rho(x,y) \cdot e^{-ik_x \cdot x} \cdot e^{-ik_y \cdot y}\, dx\, dy$$

MICHIGAN MEDICINE

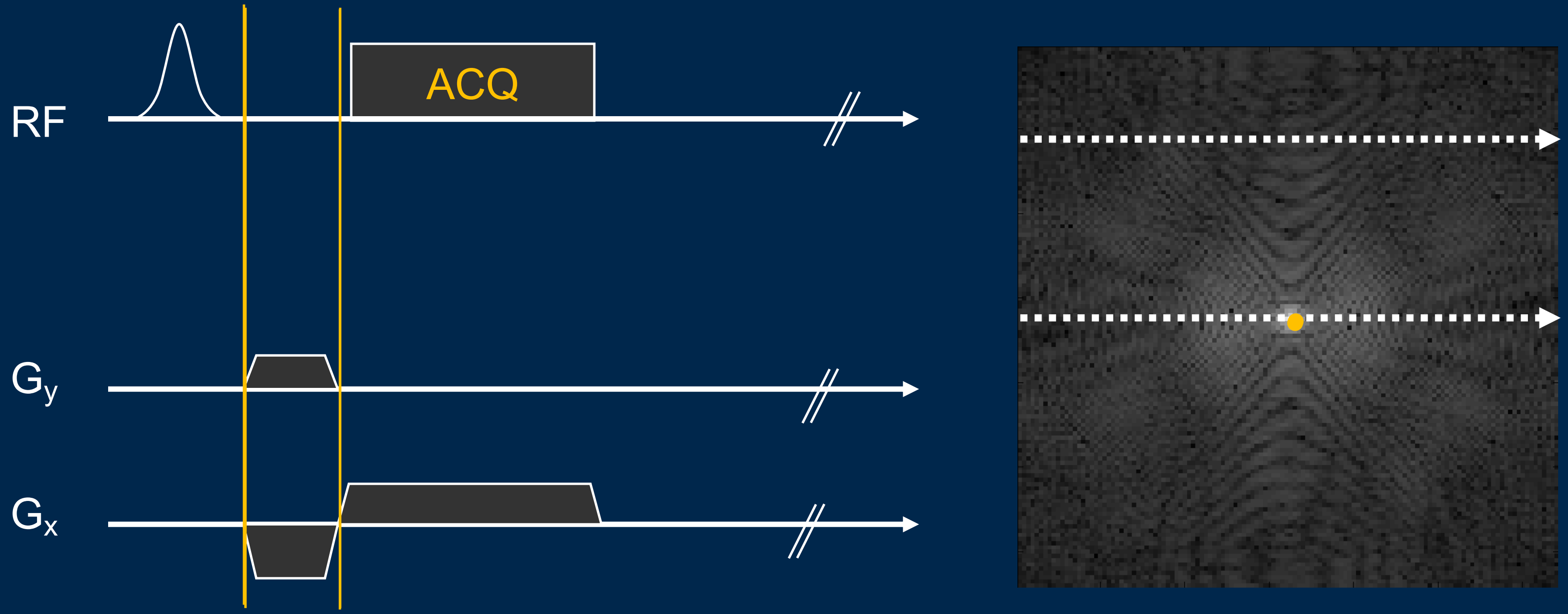
OMG, No

Signal measured in



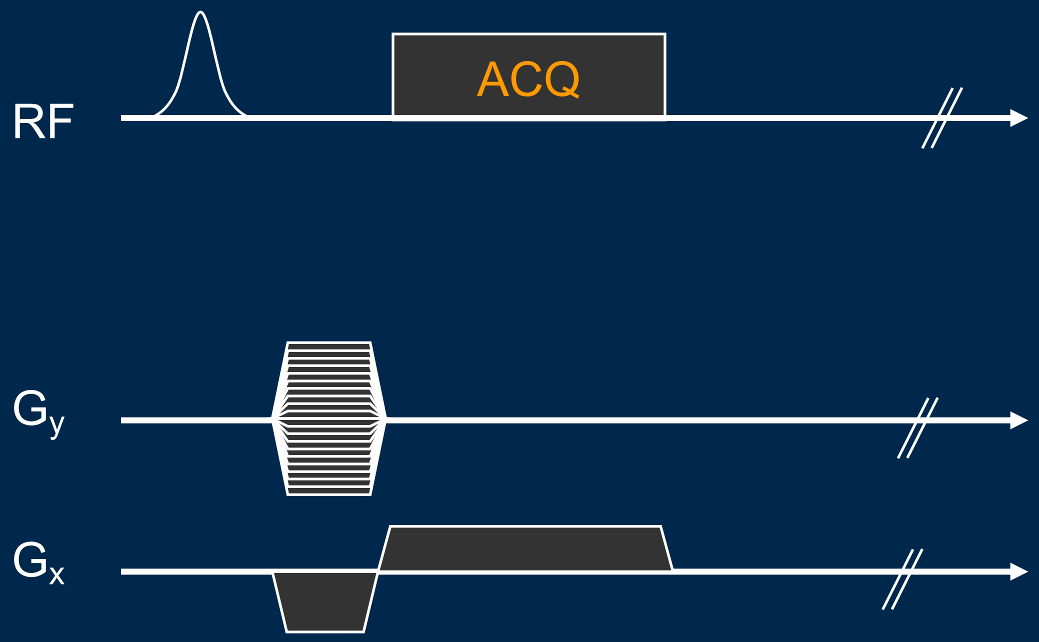
MICHIGAN MEDICINE

Traversal of k-Space with Gradients



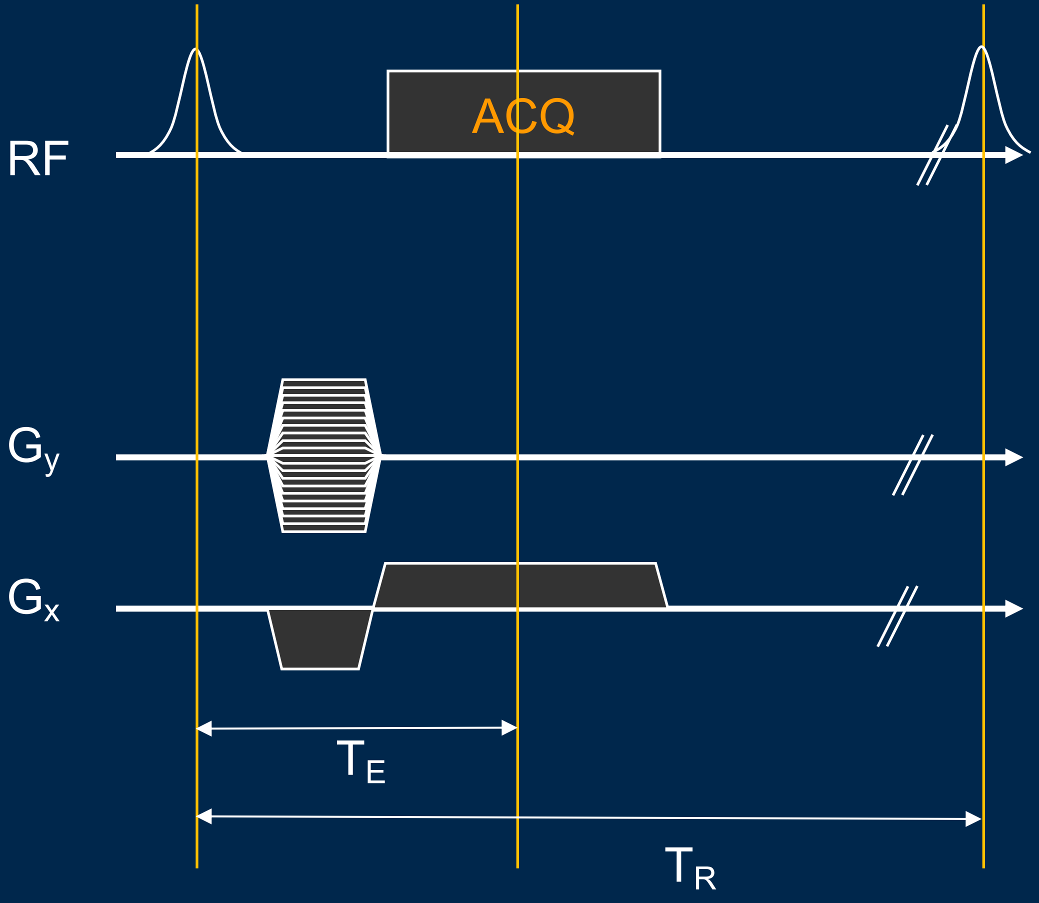
MICHIGAN MEDICINE

Traversal of K-Space with Gradients



MICHIGAN MEDICINE

Traversal of K-Space with Gradients



Fundamental Imaging Time

$$Time = T_R \cdot N_{PE} \cdot NA$$

- T_R: Amount of time needed to acquire one line (3 ms – 5 sec)
- N_{PE}: Number of lines to acquire (32 – 512)
- NA: Averages of each line (1 - ∞)

MICHIGAN MEDICINE

Summary

- Magnetic field gradients used to link precession frequency to position → spatial encoding
- Frequency encoding: gradient on during data acquisition
- Phase encoding: gradient on to accumulate phase, then
  gradient *off* during data acquisition
- Application of gradients over time dictates k-space trajectory
- The need for multiple lines of k-space data leads to long acquisition times

MICHIGAN MEDICINE