MRI Physics II: Gradients, Imaging

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Magnetic Fields in MRI

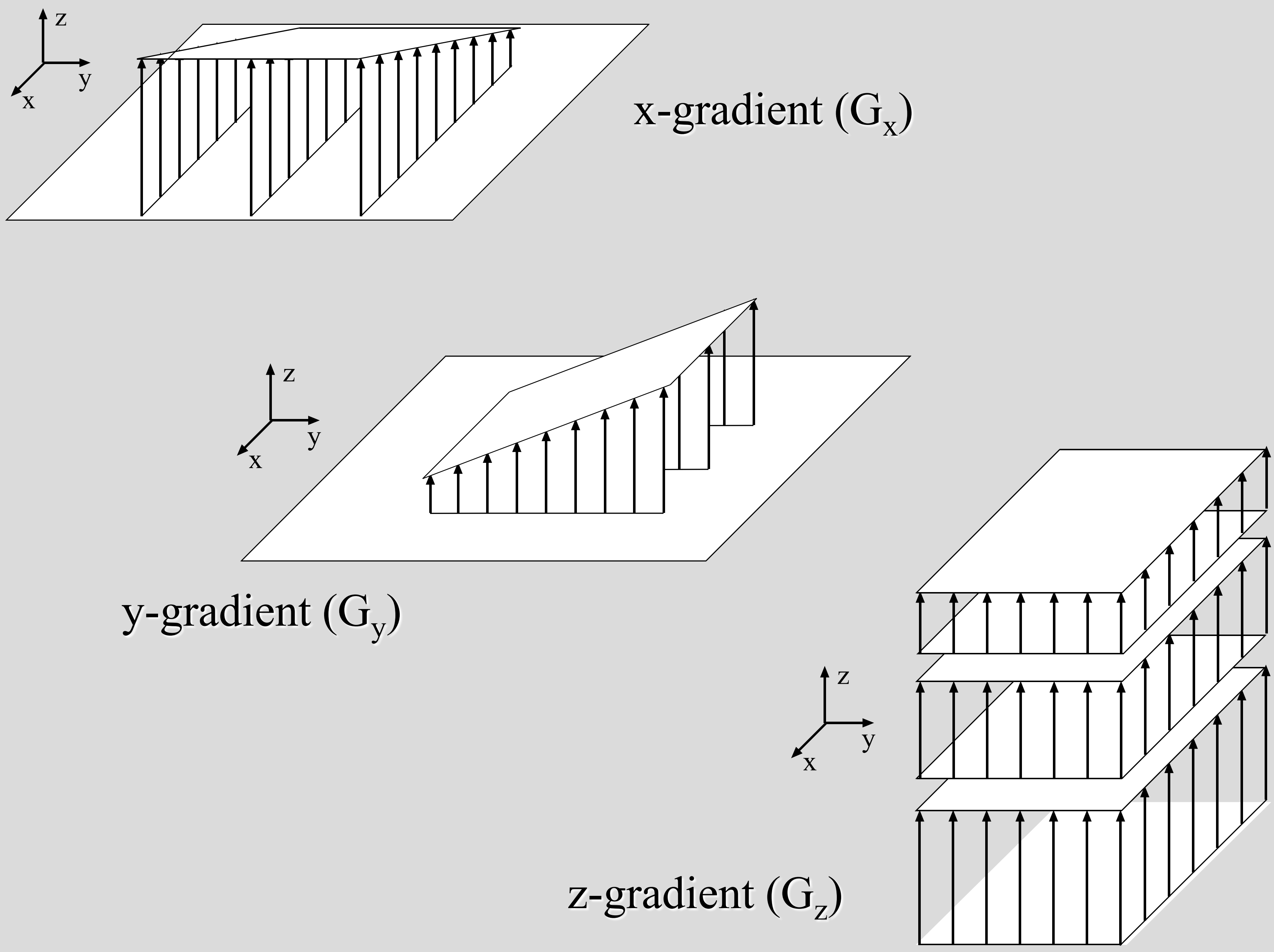
- B_0 – The main magnetic field.
  - Always on (0.5-7 T)
  - Magnetizes the object to be imaged
  - After excitation, the magnetization precesses around B_0 at $\omega_0 = \gamma B_0$
- B_1 – The rotating RF magnetic field.
  - Tips magnetization into transverse plane
  - Performs "excitation"
  - On for brief periods, then off

Gradient Fields

- The last magnetic field to be used in MRI are the gradient fields
  - 3 of them: G_x, G_y, G_z
  - These are for localization
  - Make the magnetic field different in different parts of the body, e.g. for the x-gradient:

$$B(x) = B_0 + G \cdot x$$

  - Observe the field points in the same direction as B_0 so it adds to B_0.


z
y
x
x-gradient (G_x)
z
y
x
y-gradient (G_y)
z
y
x
z-gradient (G_z)

Imaging Basics

- To understand 2D and 3D localization, we will start at the beginning with one-dimensional localization.
  - Here we “image” in 1D - the x-direction. (e.g. the L-R direction)

- We start with the simplest form of localization called “frequency encoding.”

1D Localization

- We acquire data while the x-gradient (G_x) is turned on and has a constant strength.
- Recall that a gradient makes the strength of the magnetic field vary in a particular direction.
- In this case, having a positive x-gradient implies that the farther we move along in the x-direction (e.g. the farther right we move) the magnetic field will increase.

$$B(x) = B_0 + G \cdot x$$

Frequency Encoding

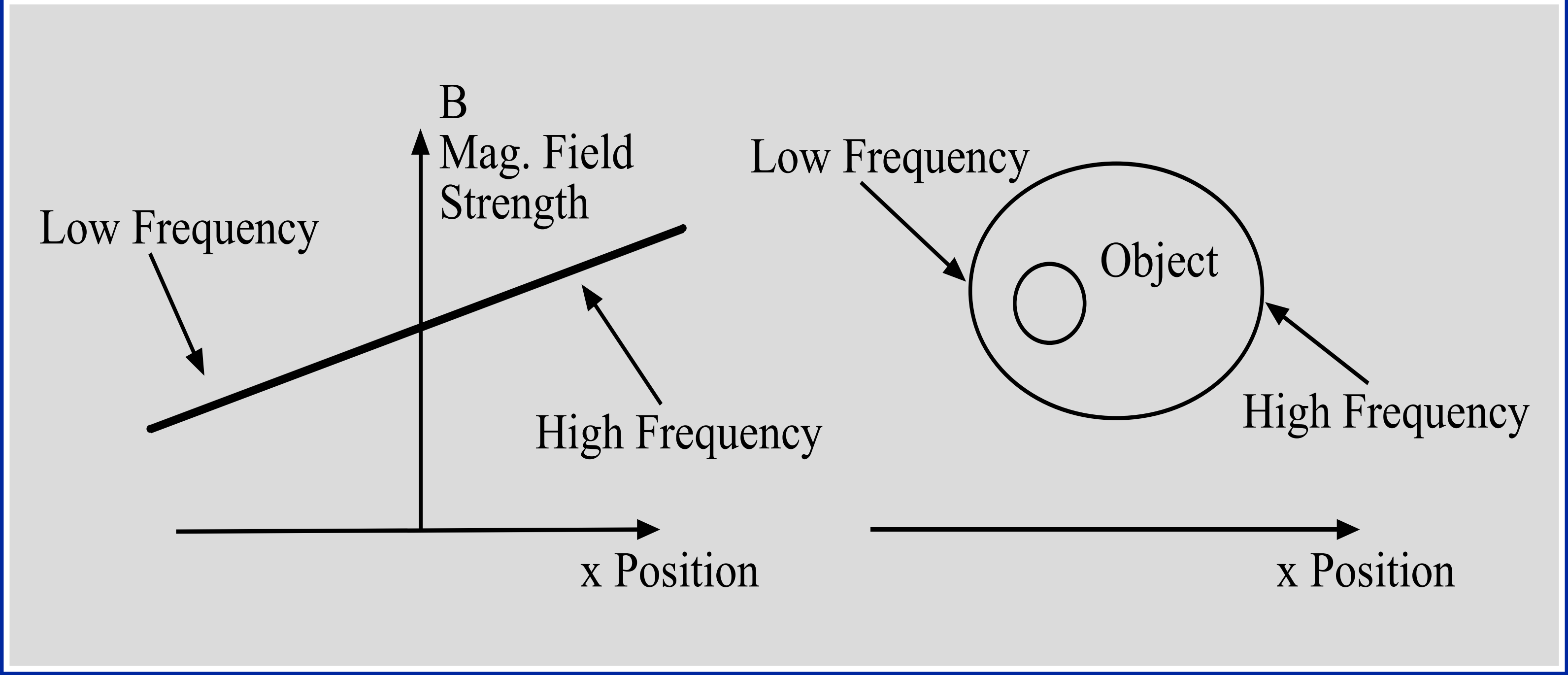
- A fundamental property of nuclear spins says that the frequency at which they precess (or emit signals) is proportional to the magnetic field strength:

$$\omega = \gamma B$$ - The Larmor Relationship

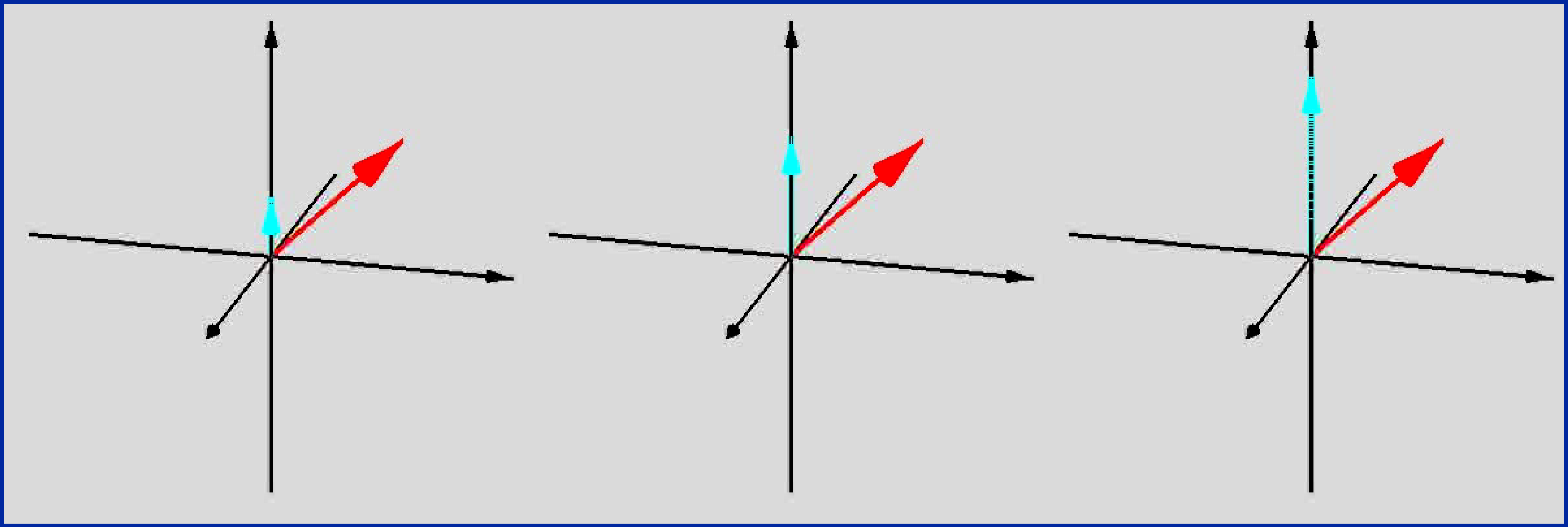
- This says that precession frequency now increases as we move along the x-direction (e.g. as we move rightwards).

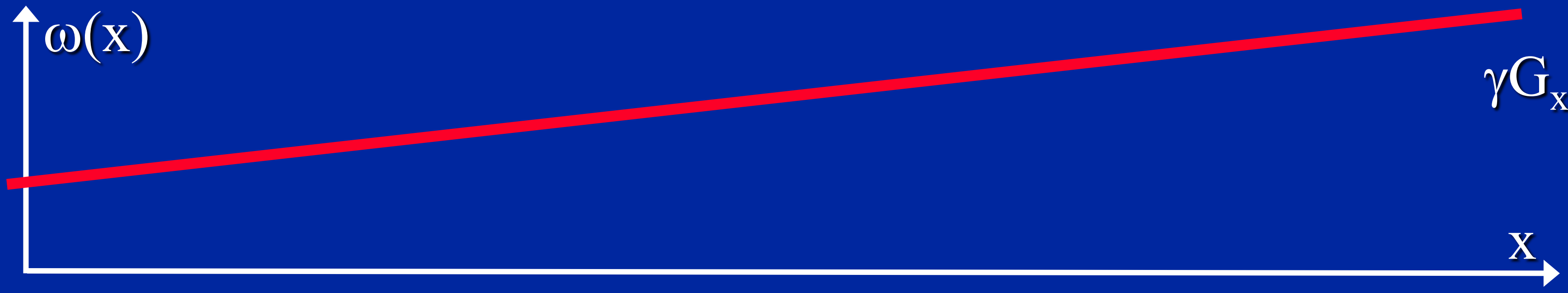
$$\omega(x) = \gamma\ (B_0 + G \cdot x).$$

Frequency Encoding



Spins in a Magnetic Field





The Fourier Transform

- The last part of this story is the Fourier transform.
- A function of time is made up of a sum of sines and cosines of different frequencies.
- We can break it down into those frequency components

The Fourier Transform .com

$$\mathcal{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t)e^{-i2\pi ft}\,dt$$

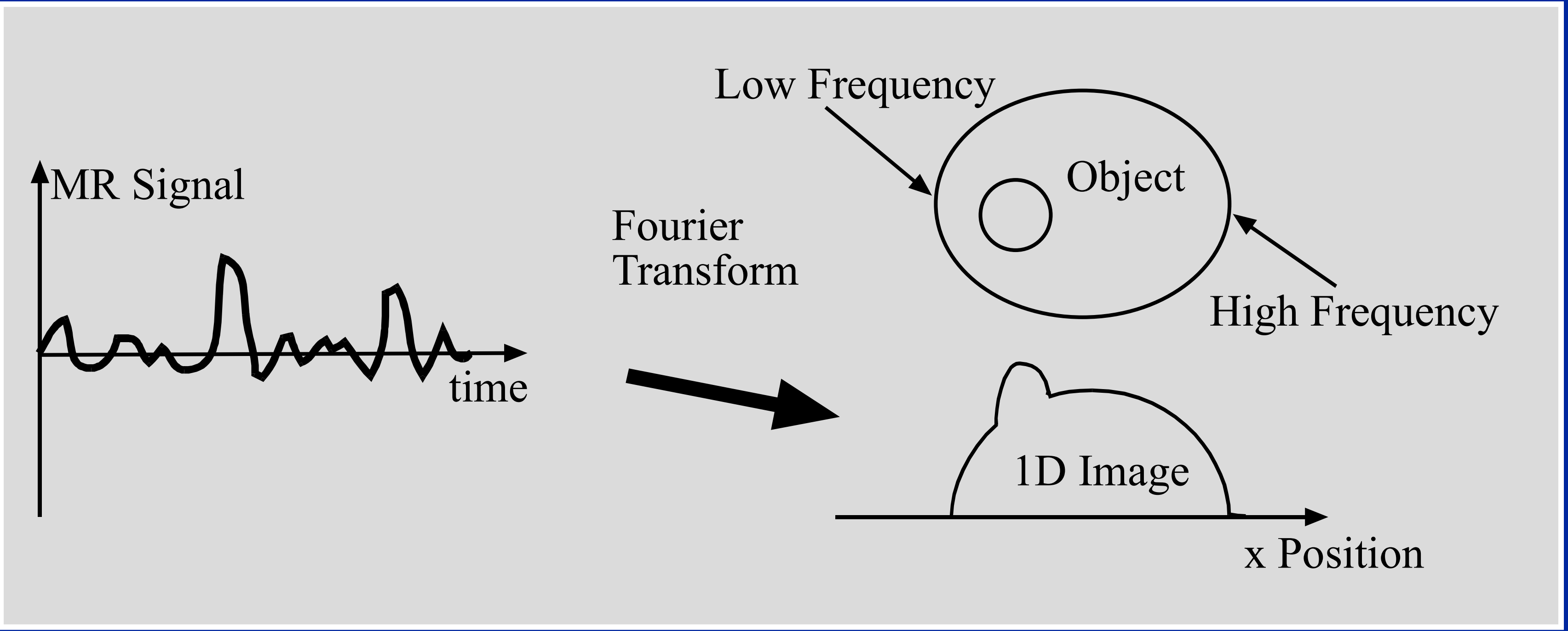
$$\mathcal{F}^{-1}\{G(f)\} = g(t) = \int_{-\infty}^{\infty} G(f)e^{i2\pi ft}\,df$$

Recall that $e^{i2\pi ft} = \cos(2\pi ft) + i\sin(2\pi ft)$

Fourier Transforms

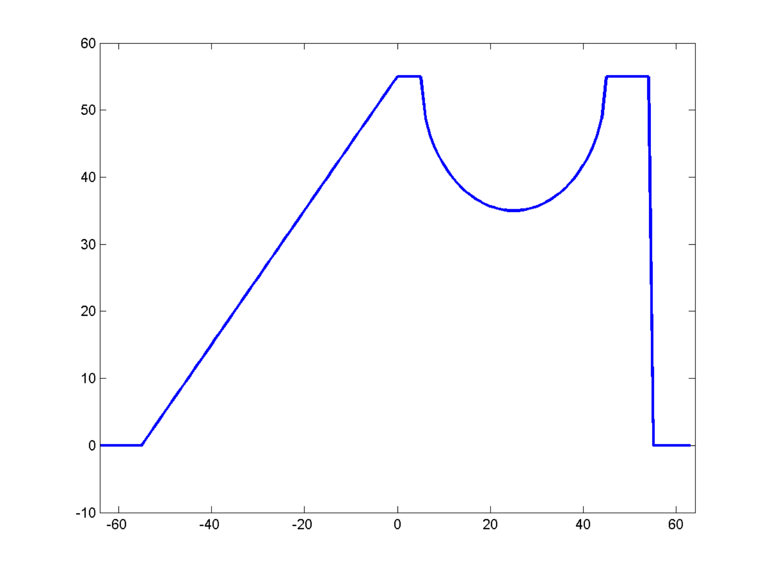
- In short, the Fourier transform is the mathematical operator (computer program) that breaks down each MR signal into its frequency components.
- If we plot the strength of each frequency, it will form a representation (or image) of the object in one-dimension.

Fourier Transforms

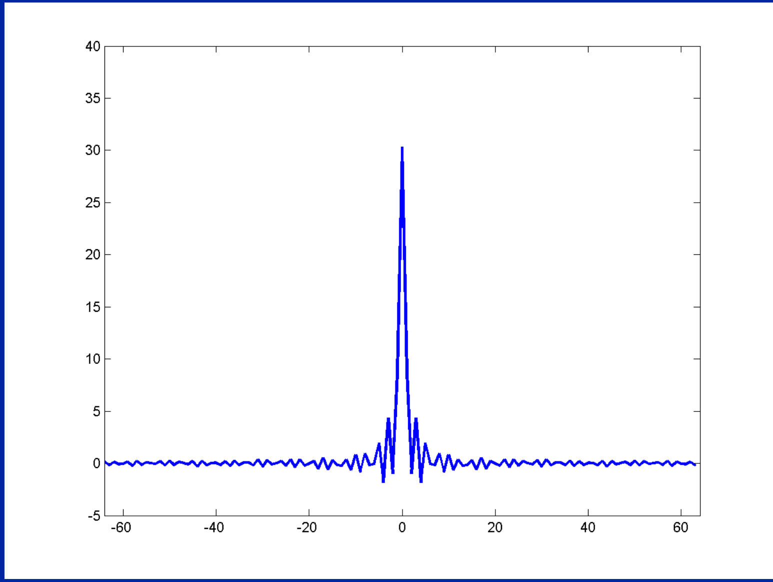


Fourier Representation of Images

- Decomposition of images into frequency components, e.g. into sines and cosines.



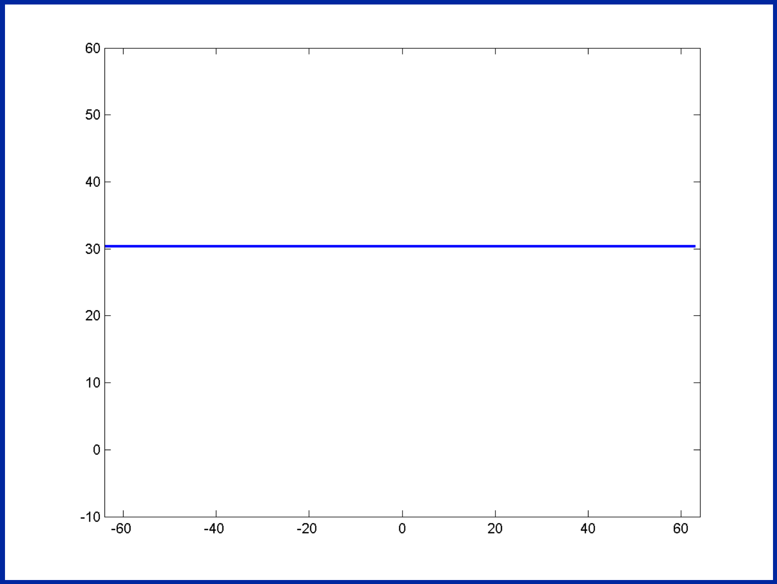
1D Object

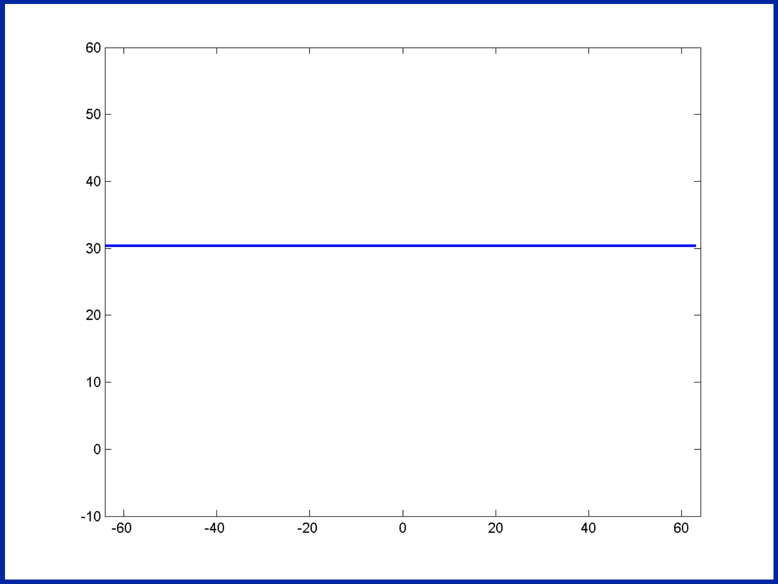


Fourier Data

1D Fourier Transform

0^{th} Frequency Component



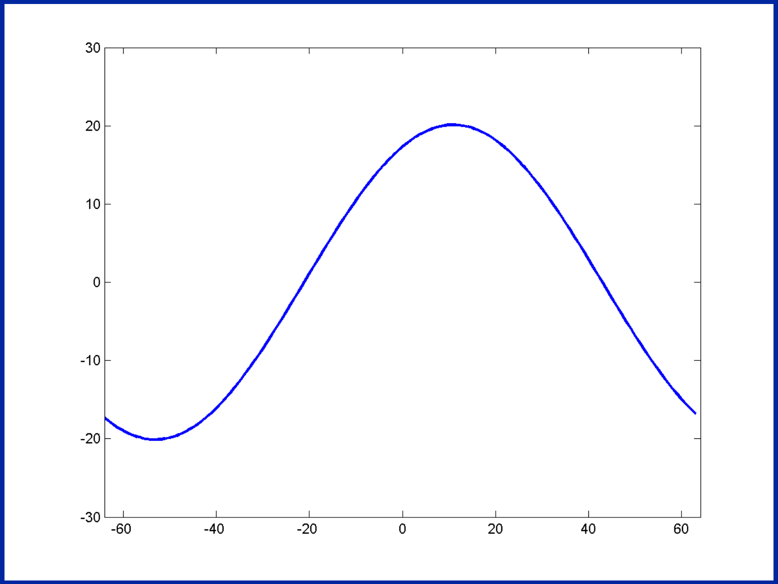


New Components

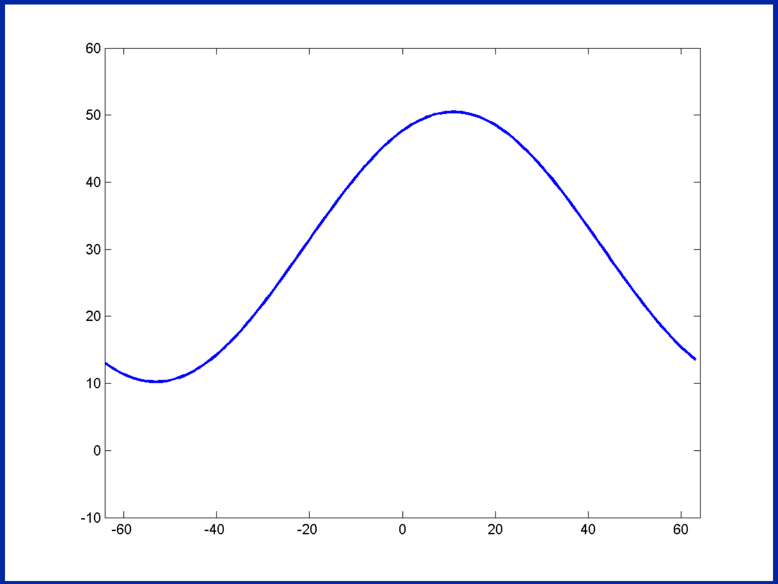
Cumulative Sum
of Components

1D Fourier Transform

1st Frequency Component



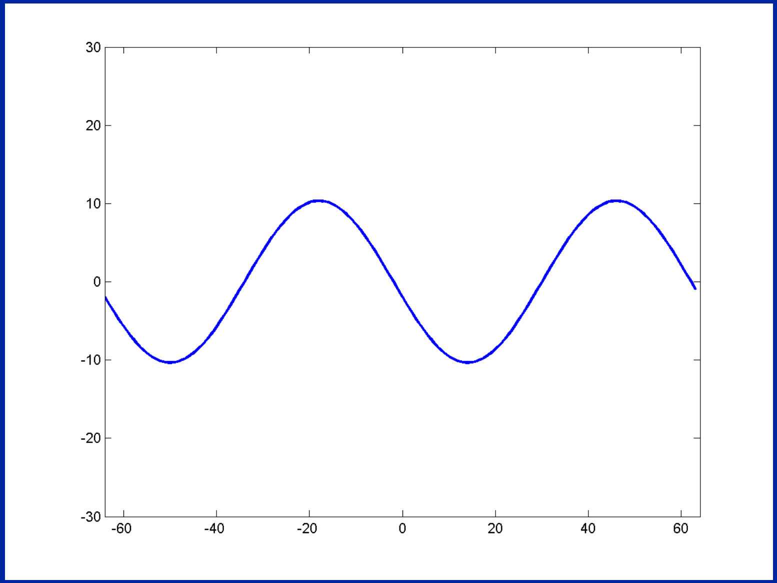
New Components



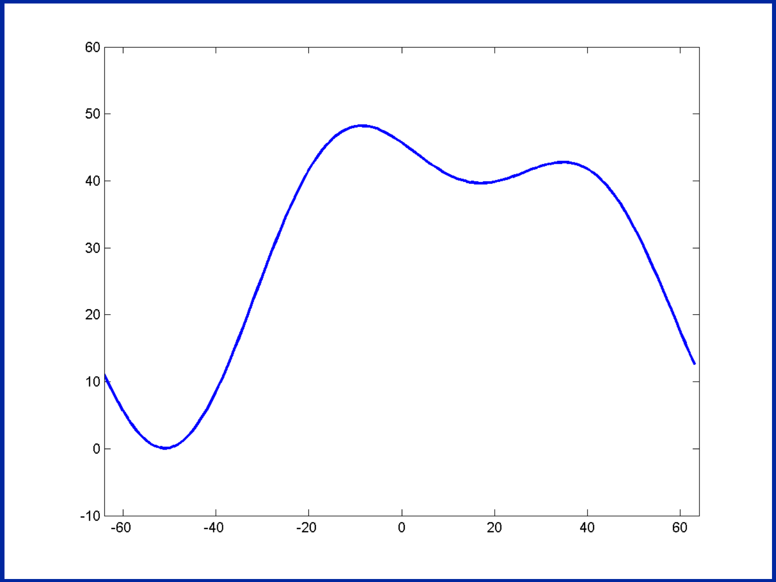
Cumulative Sum of Components

1D Fourier Transform

2nd Frequency Component



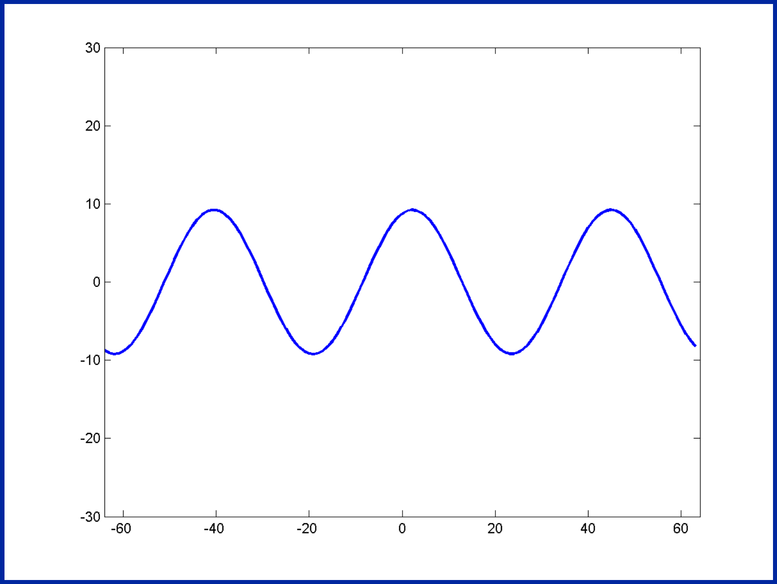
New Components



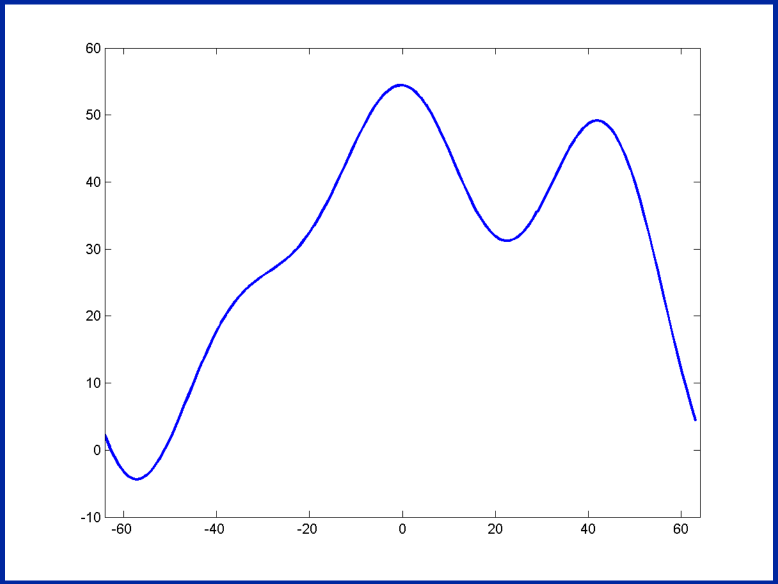
Cumulative Sum of Components

1D Fourier Transform

3rd Frequency Component



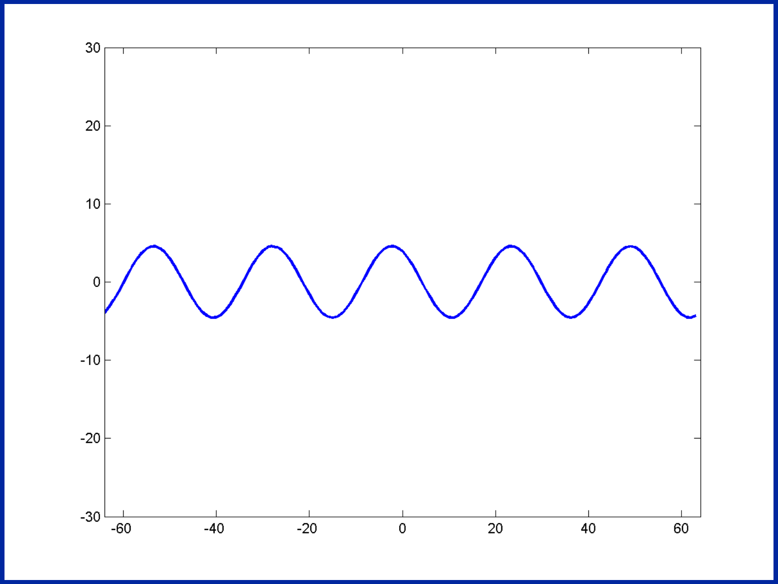
New Components



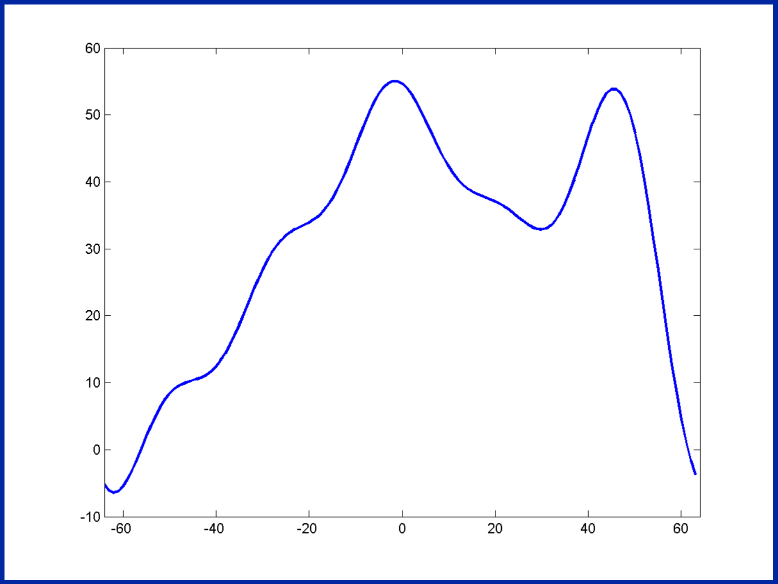
Cumulative Sum of Components

1D Fourier Transform

5th Frequency Component



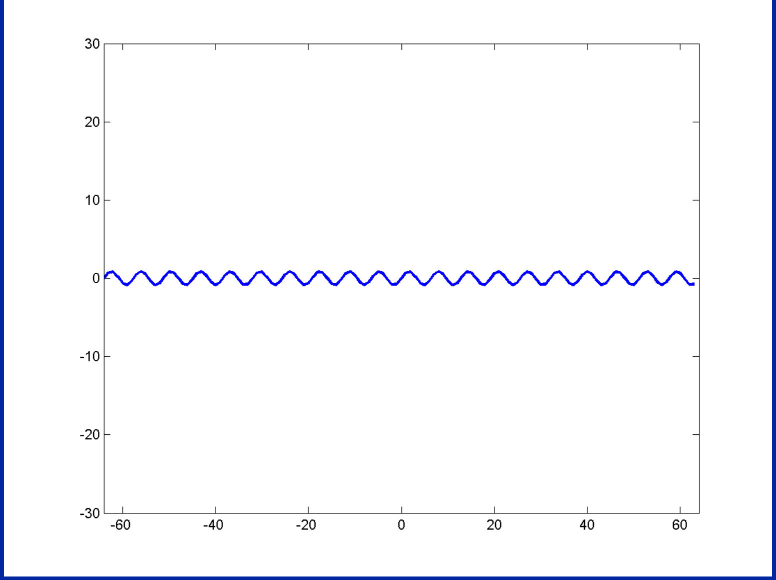
New Components



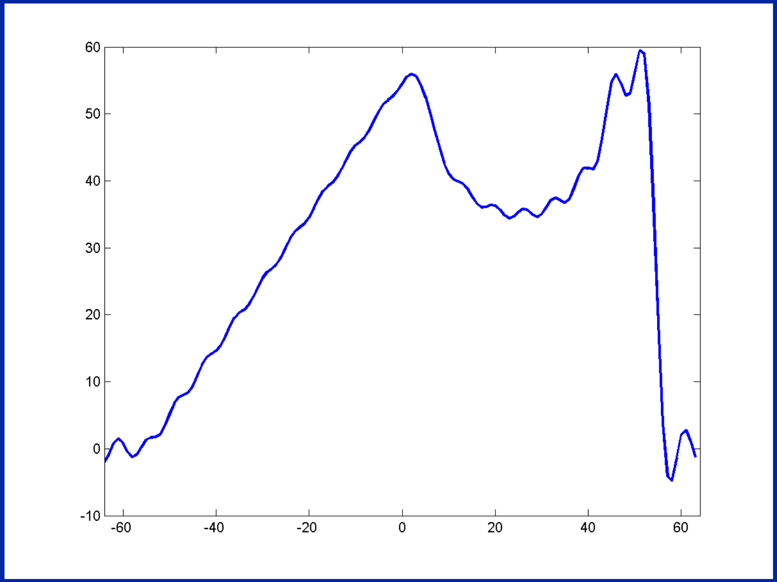
Cumulative Sum
of Components

1D Fourier Transform

20th Frequency Component



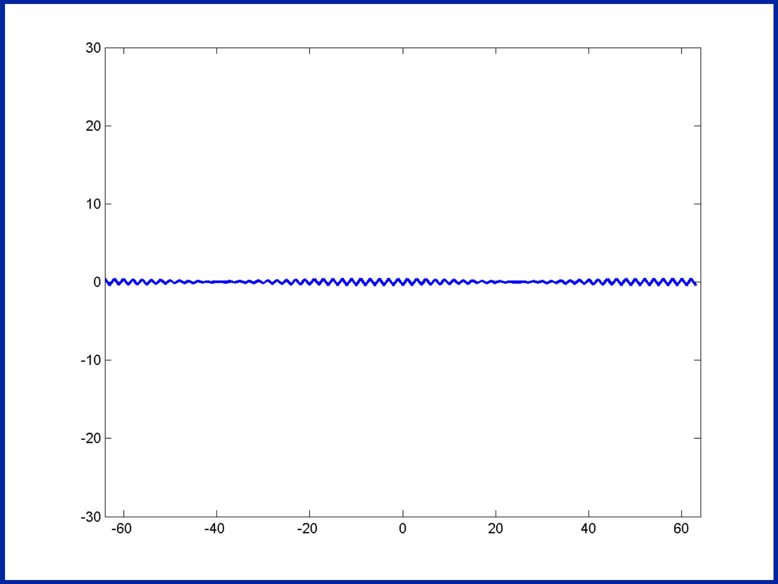
New Components



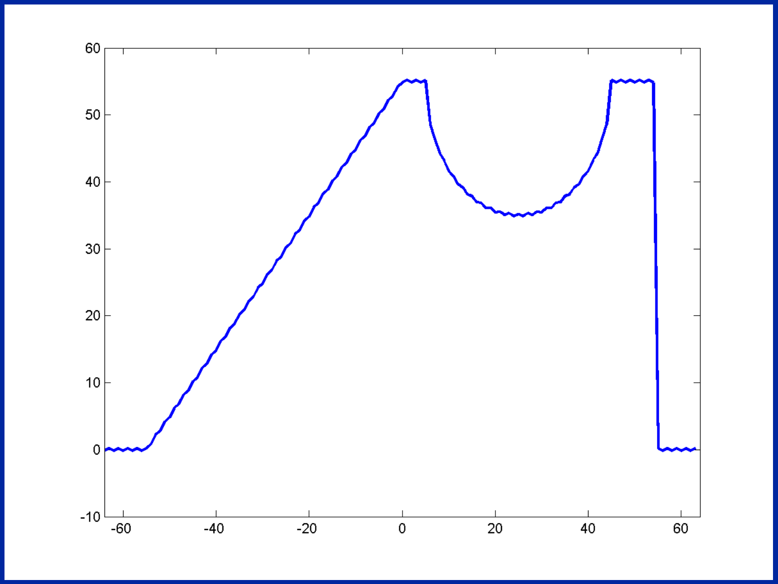
Cumulative Sum of Components

1D Fourier Transform

63rd Frequency Component



New Components



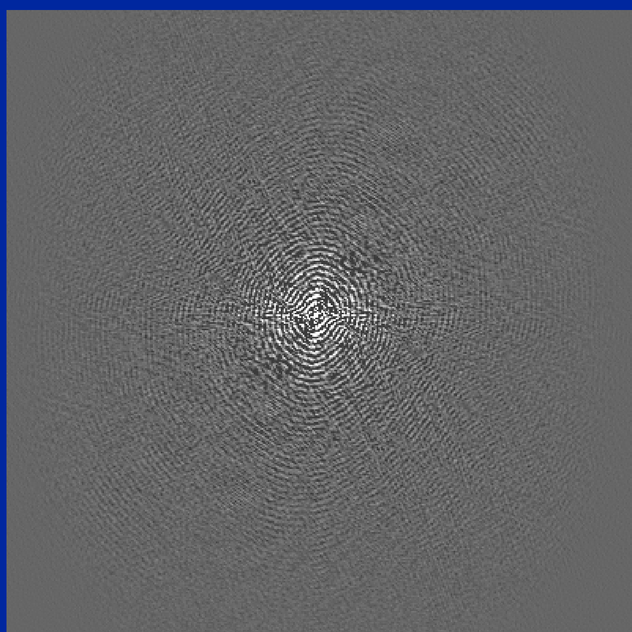
Cumulative Sum of Components

Fourier Acquisition

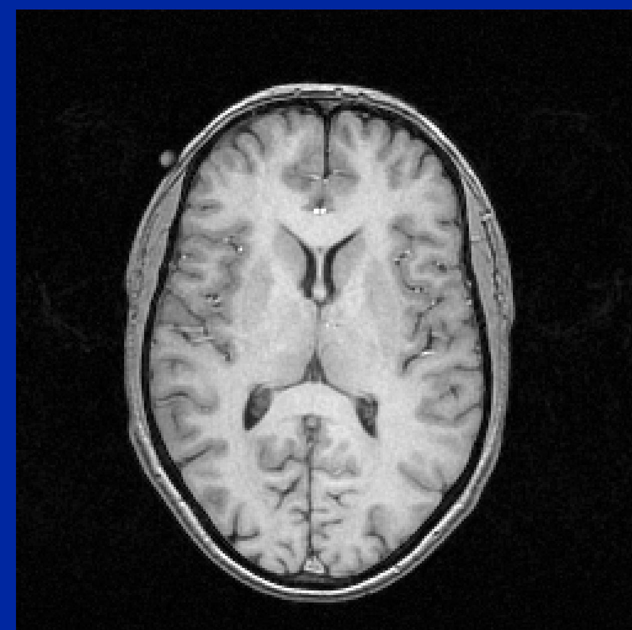
- In MRI, we are acquiring Fourier components
  - Then…we take the FT of the acquired data to create an image
- The more Fourier components we acquire, the better the representation

2D Imaging - 2D Fourier Transform

In MRI, we are acquiring Fourier components
– works in two dimensions as well



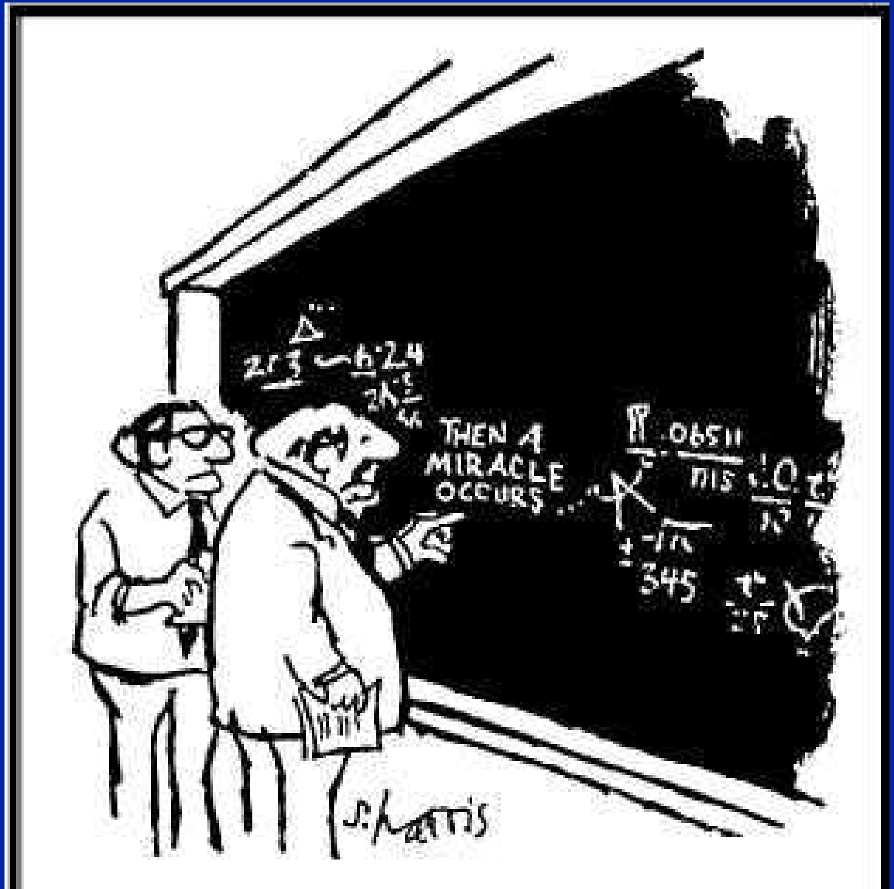
2D FFT ↔



Acquired Data

Resultant Image

There was a pretty big leap here…



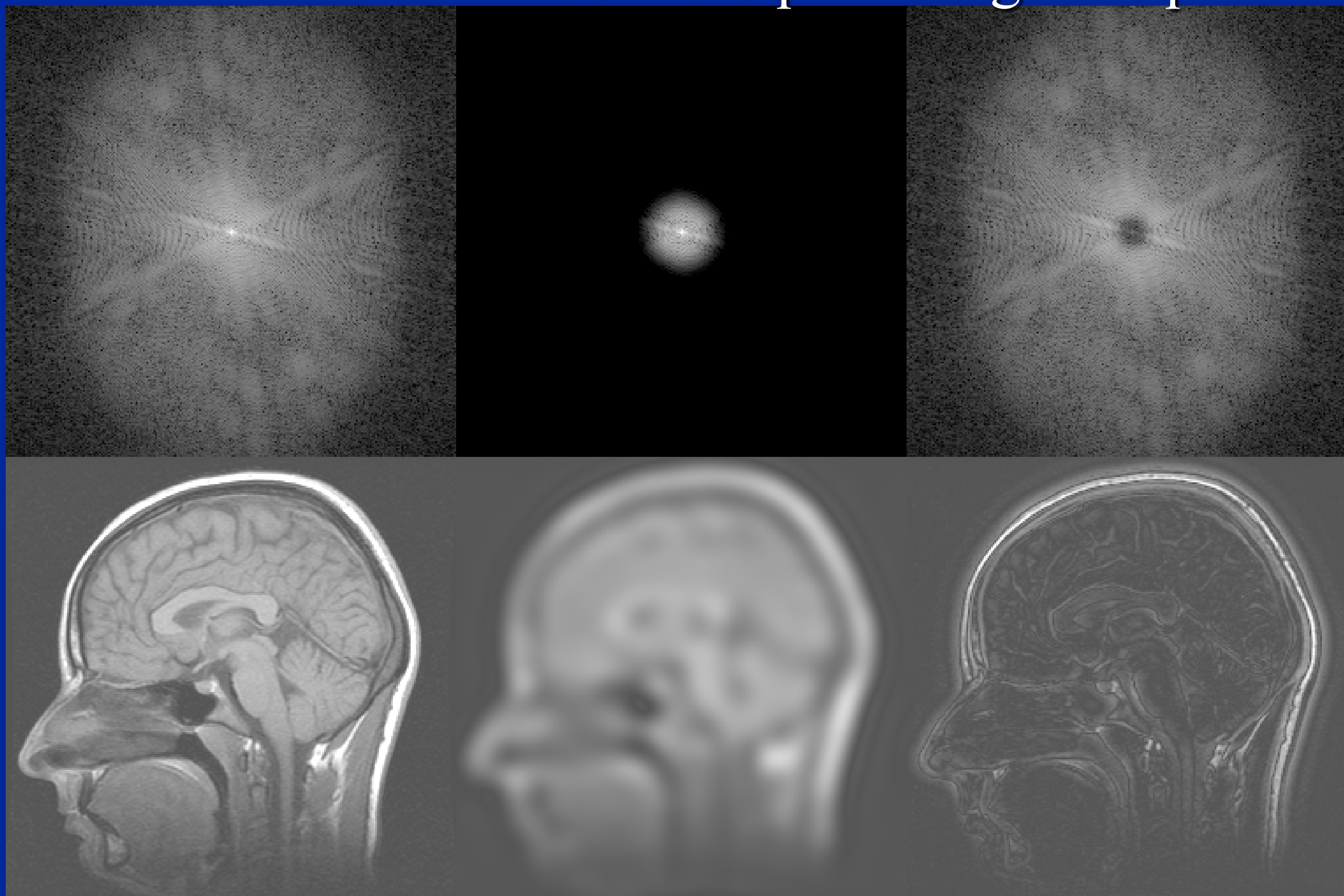
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Spatial Frequencies in 2D

Full Low Freq High Freq

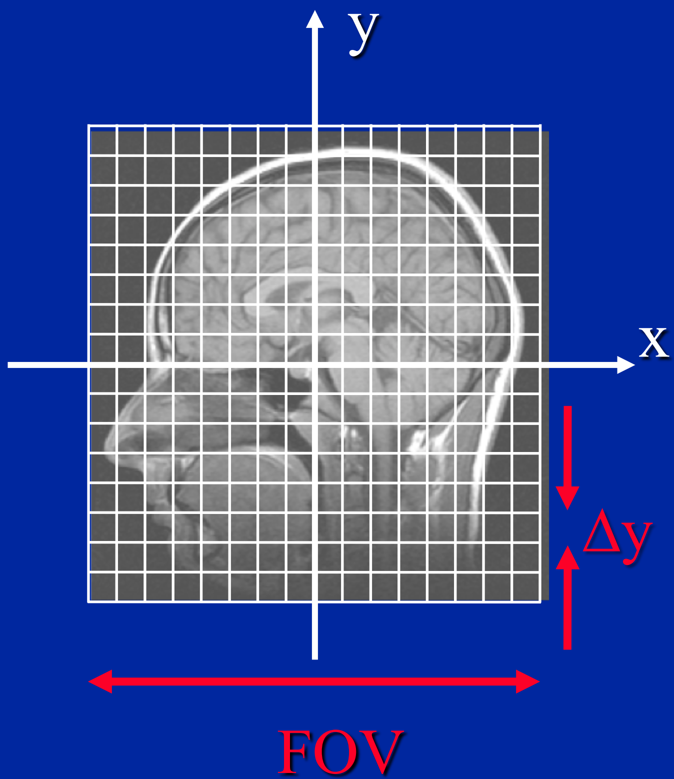
Fourier Data

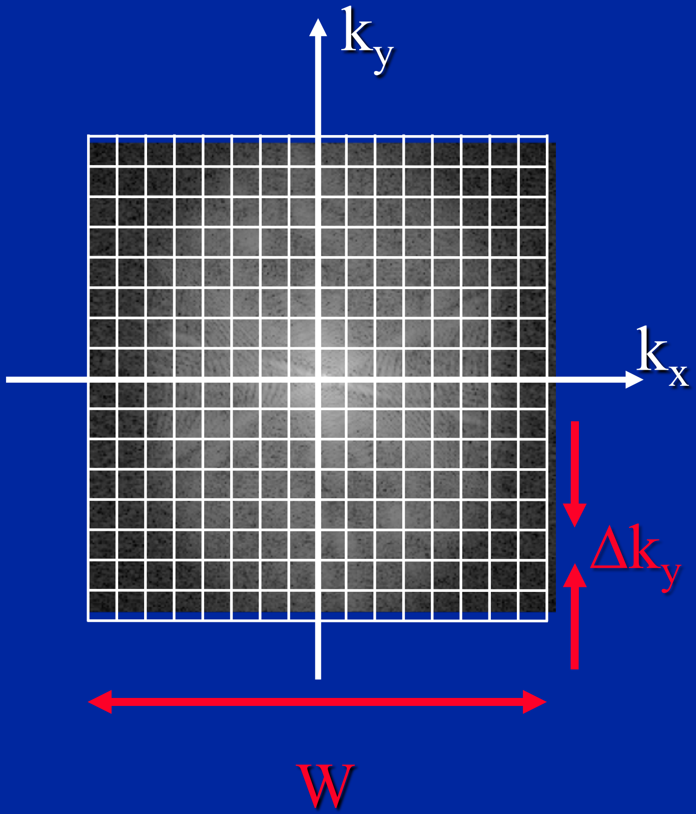
Image Data



Low Res (contrast) Edges

Resolution and Field of View



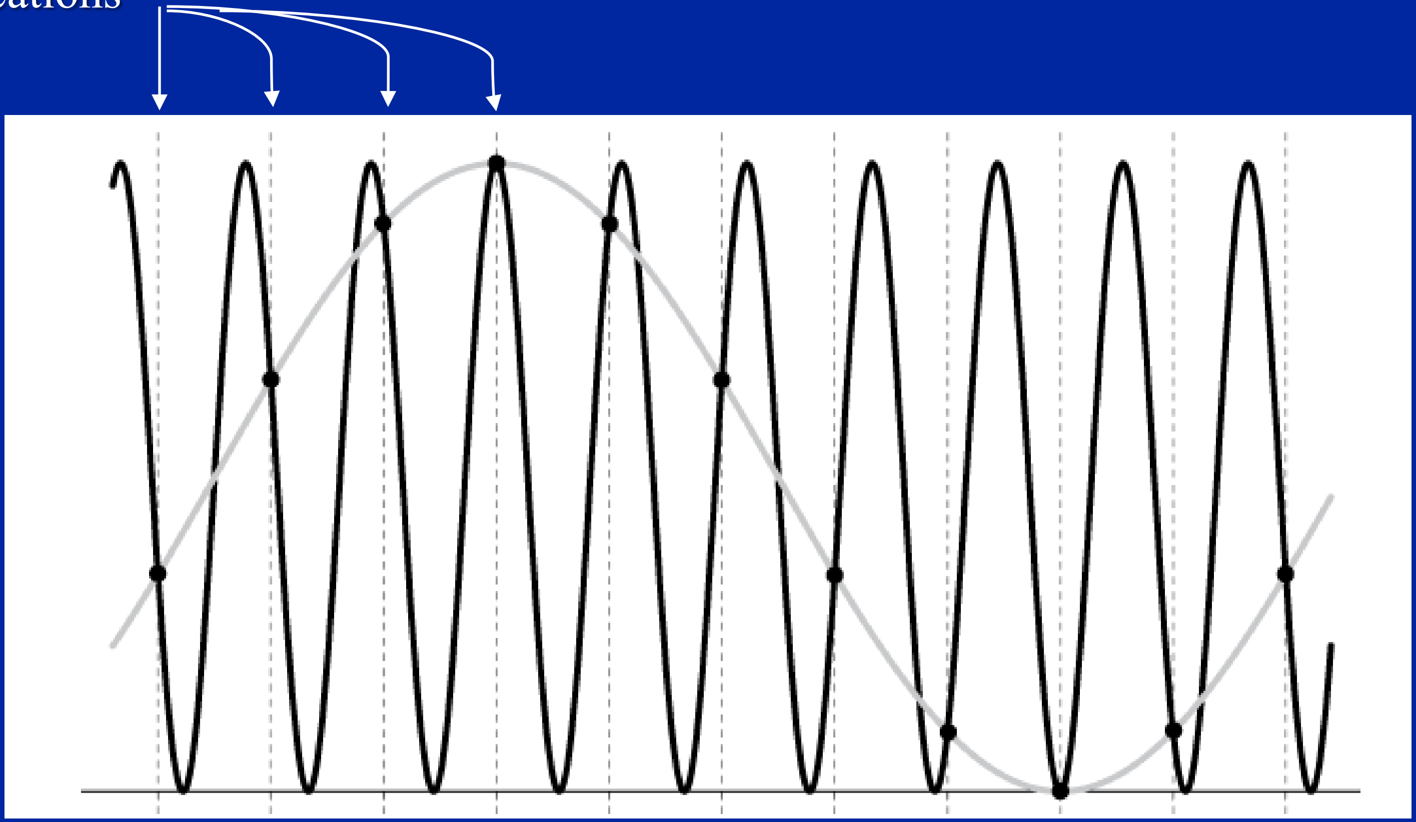


- Field of view is determined by spacing of samples: $FOV = 1 / \Delta k$
- Resolution is determined by size of the area acquired: $\Delta x = 1 / W$

Courtesy Luis Hernandez

Aliasing

Sample
Locations

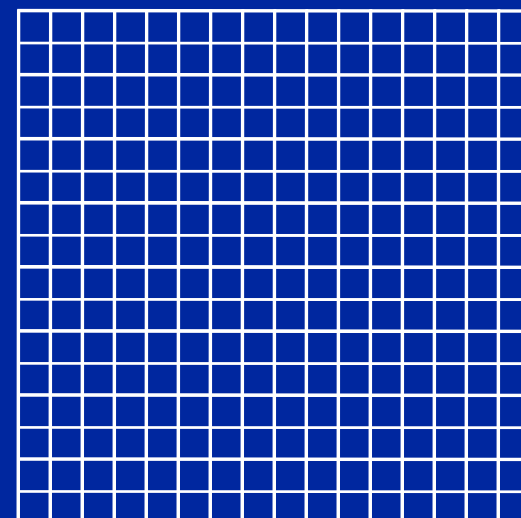


Notice how two different frequencies have the same samples?
(won't have this problem if you sample more finely)

Courtesy Luis Hernandez

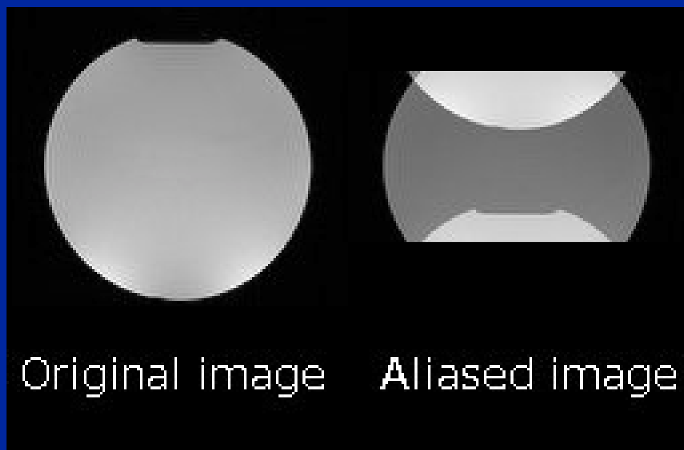
Aliasing

K space

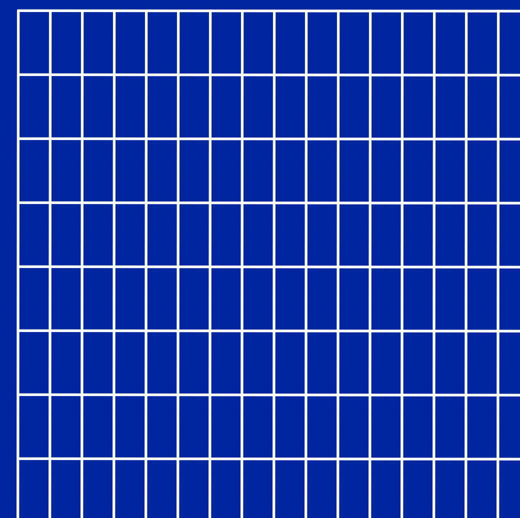


Courtesy Luis Hernandez

Aliasing



K space



(Collected only every other line)

If you don't sample finely enough, higher frequencies look like (take on the alias of a) lower frequencies

Courtesy Luis Hernandez

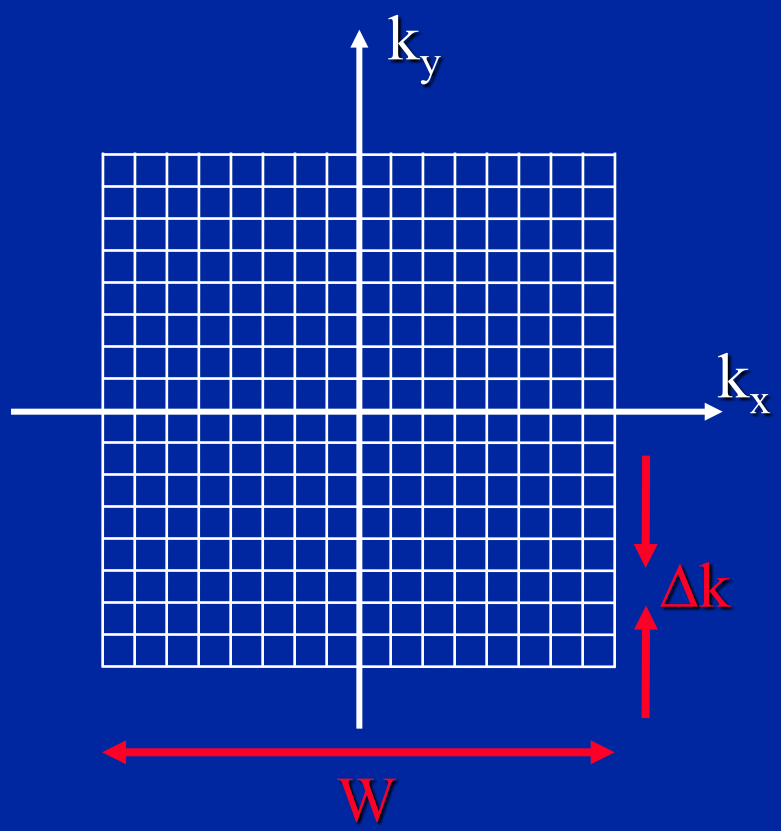
Resolution and Field of View

Resolution is determined by size of the area acquired:

$$\Delta x = 1 / W$$

Field of view is determined by spacing of samples:

$$FOV = 1 / \Delta k$$



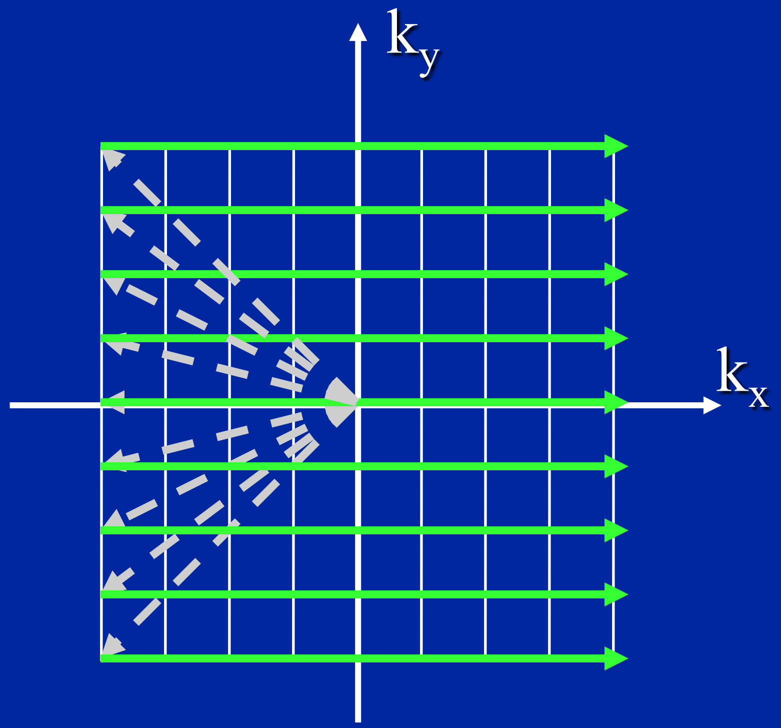
Goals of Image Acquisition

- Acquire 2D (or 3D) Fourier data
- Acquire samples finely enough to prevent aliasing (FOV)
- Acquire enough samples for the desired spatial resolution (Δx)
- Acquire images with the right contrast
- Do it fast as possible
- Do it without distortions and other artifacts

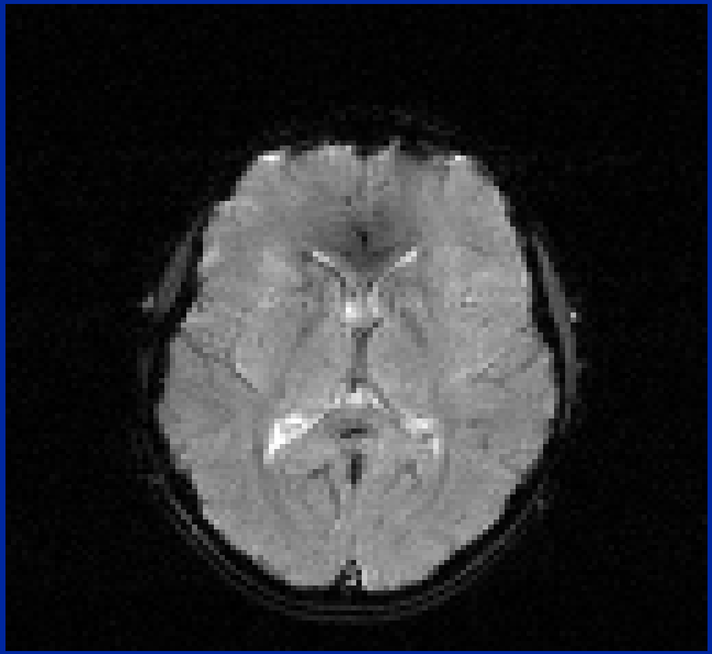
Some Common Imaging Methods

- Conventional (spin-warp) Imaging
- Echo Planar Imaging (EPI)
- Spiral Imaging

Conventional (Spin-Warp) Imaging

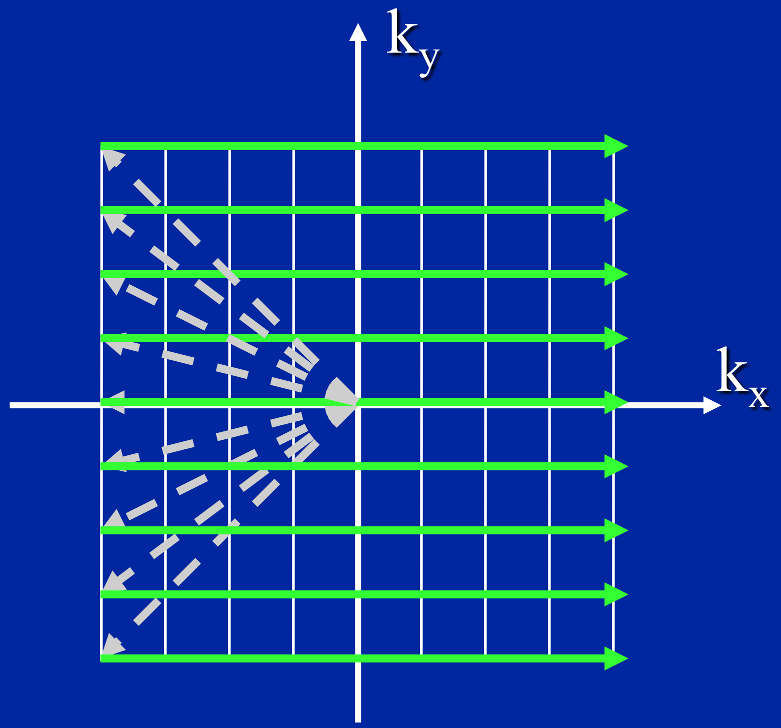


One Line at a Time



128x128 FLASH/SPGR
TR/TE/flip = 50ms/30ms/30º
0.2 slices per sec, single slice for fMRI

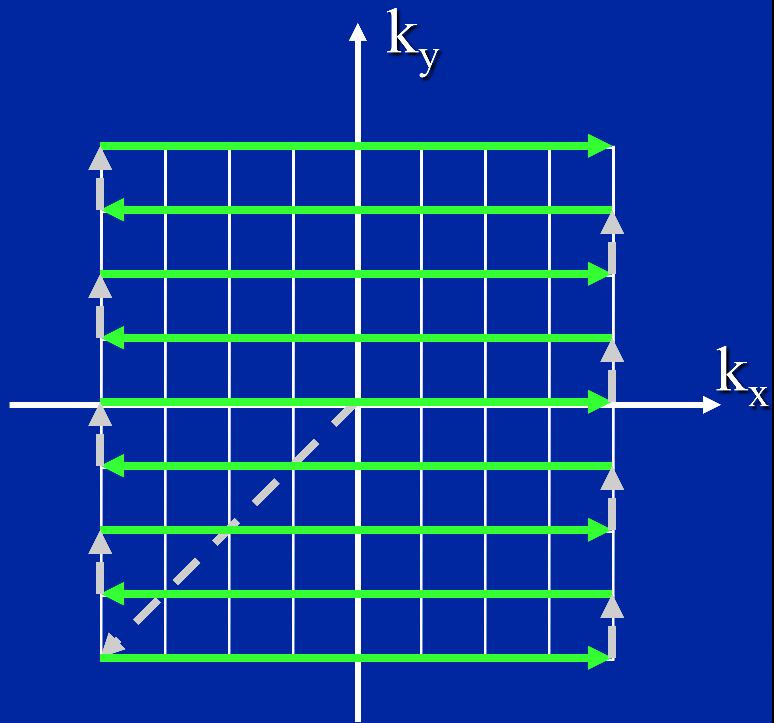
Conventional (Spin-Warp) Imaging



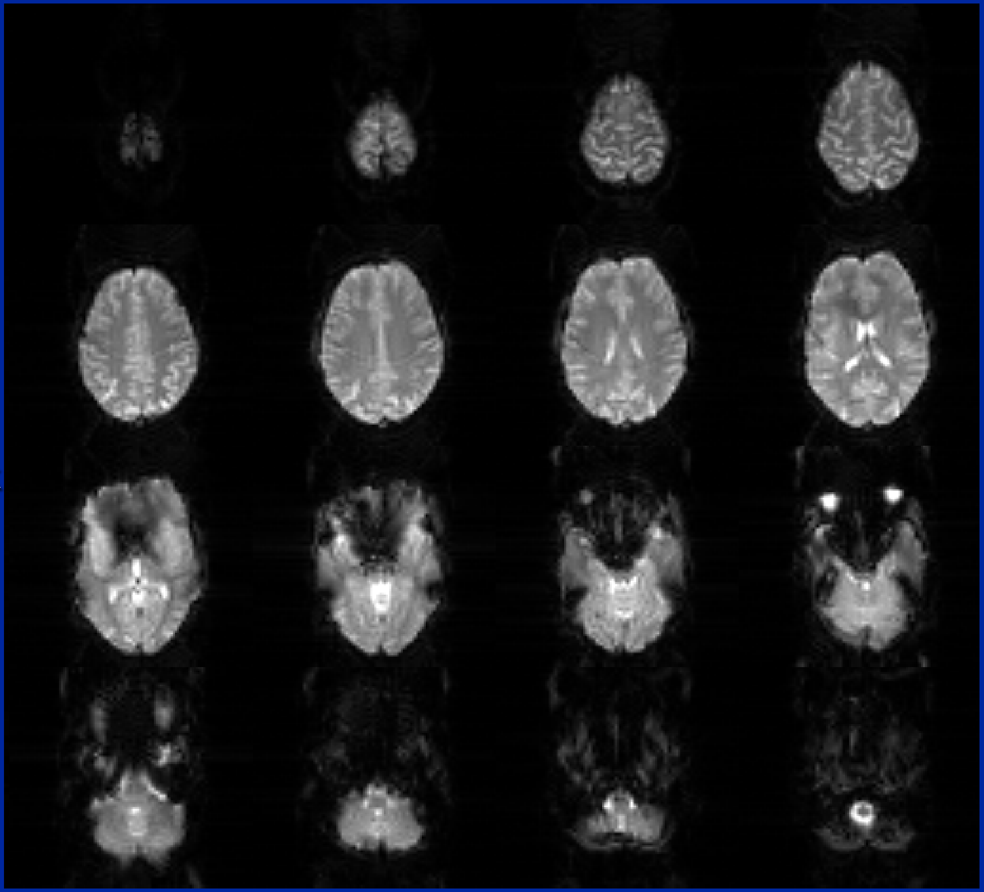
One Line at a Time

- Known as:
  - GRE, FLASH, SPGR
- Typically matrix sizes for fMRI
  - 128x64, 128x128
- Acquisition rates
  - 3-10 sec/image
  - 1-4 slices
- Usually best for structural imaging

Echo Planar Imaging (EPI)

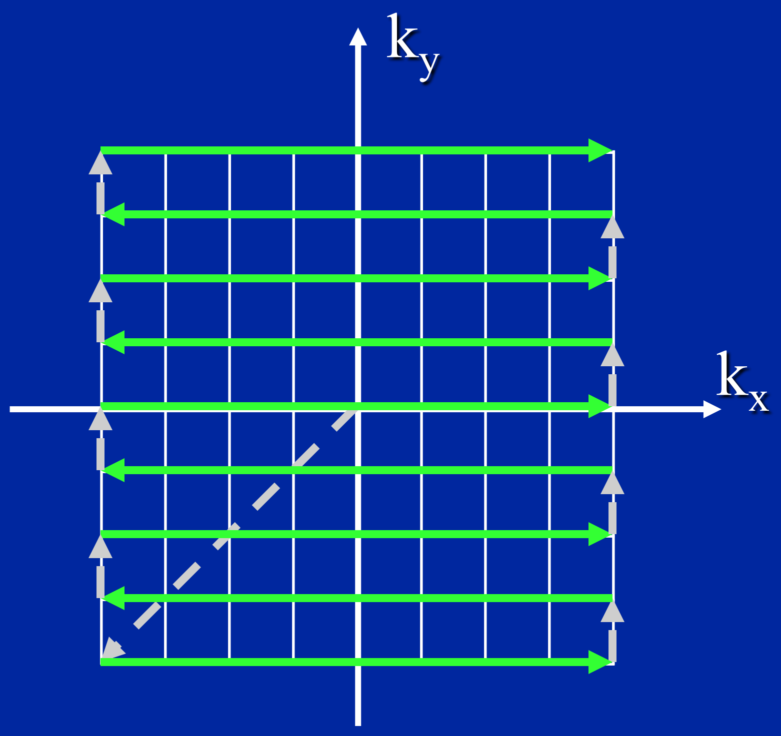


Zig-Zag Pattern



Single-shot EPI, TE = 40 ms, TR = 2 s, 20 slices

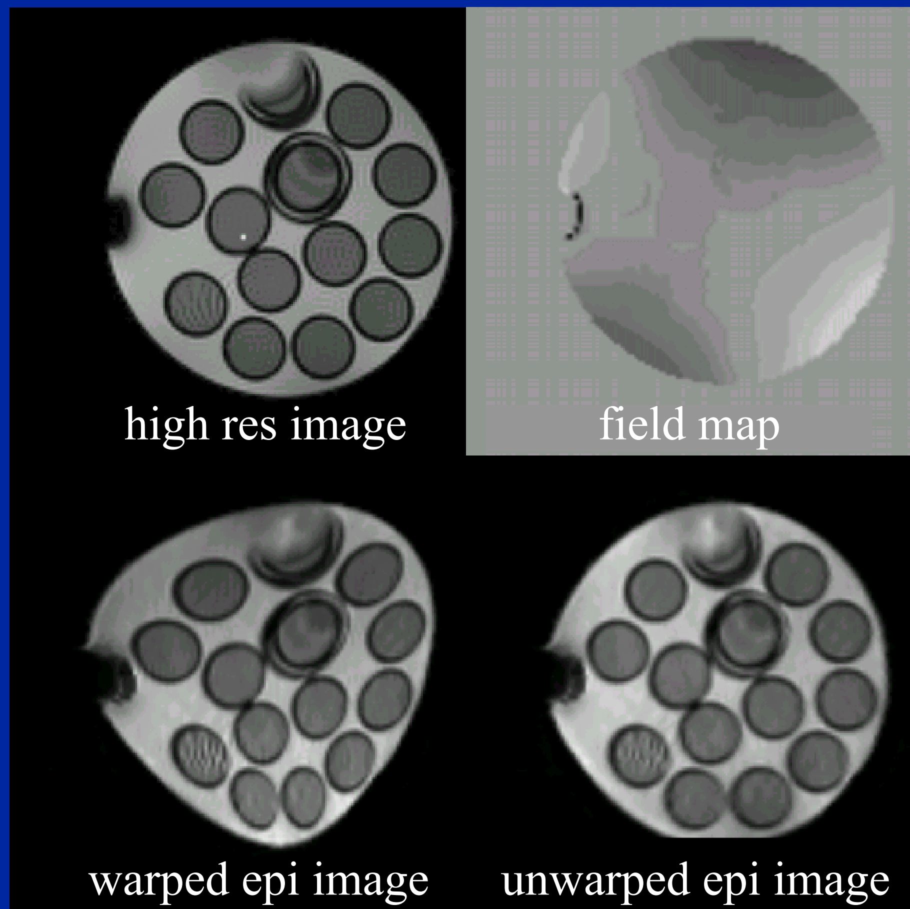
Echo Planar Imaging (EPI)



Zig-Zag Pattern

- Single-shot acquisition
- Typically matrix sizes for fMRI
  - 64x64, 96x96
  - 128x128 interleaved
- Acquisition rates
  - TR = 1-2 sec
  - 20-30 slices
- Suffers some artifacts
  - Distortion, ghosts

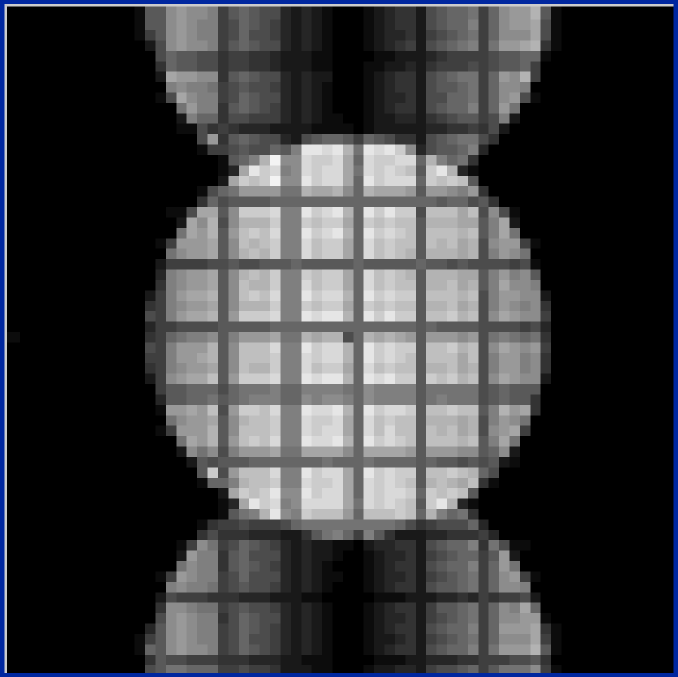
EPI Geometric Distortions

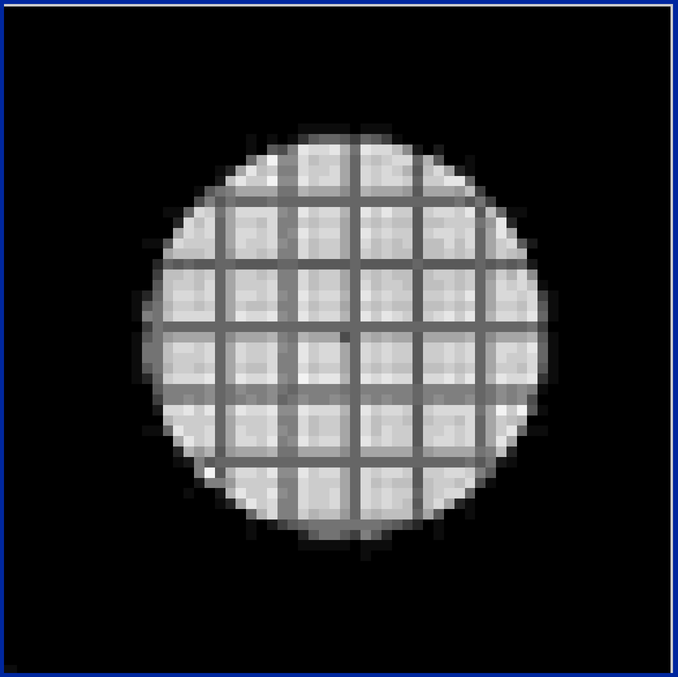


Jezzard and Balaban, MRM 34:65-73 1995

Courtesy of P. Jezzard

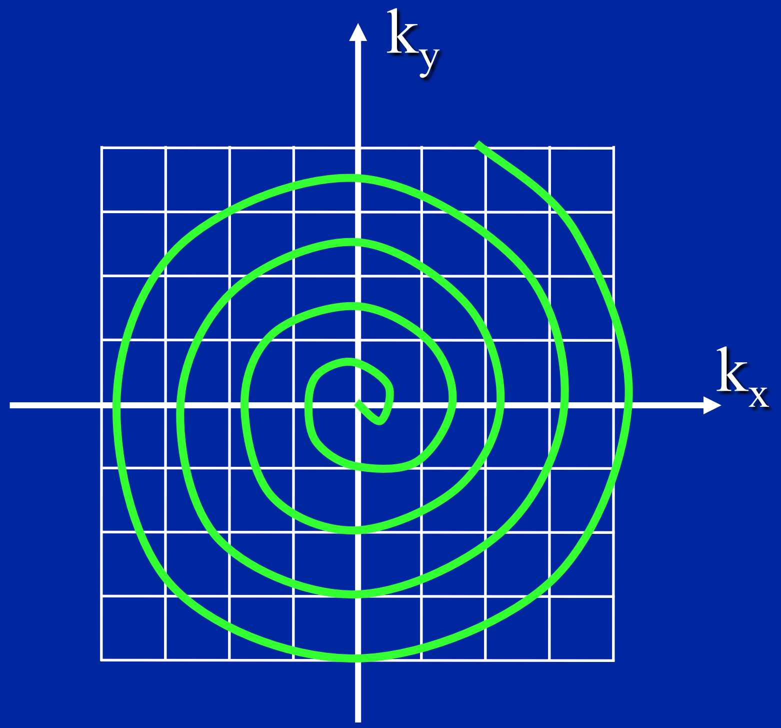
EPI Nyquist Ghost



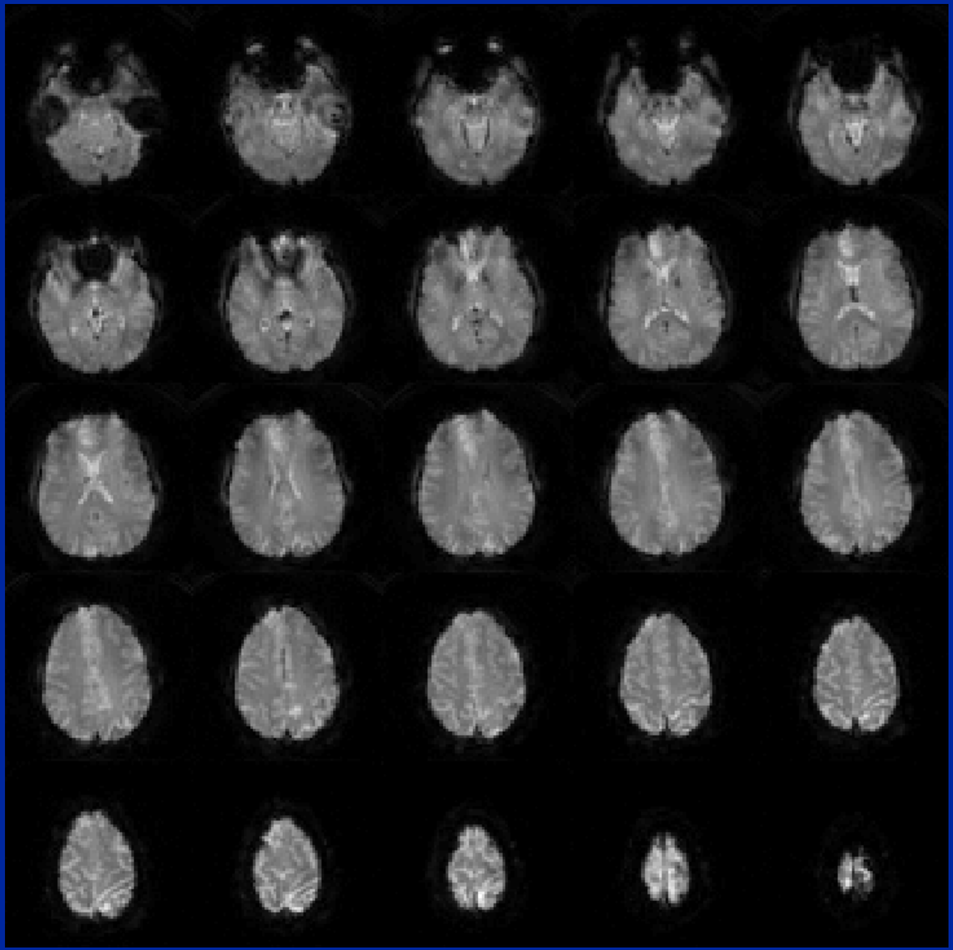


Courtesy of P. Jezzard

Spiral Imaging

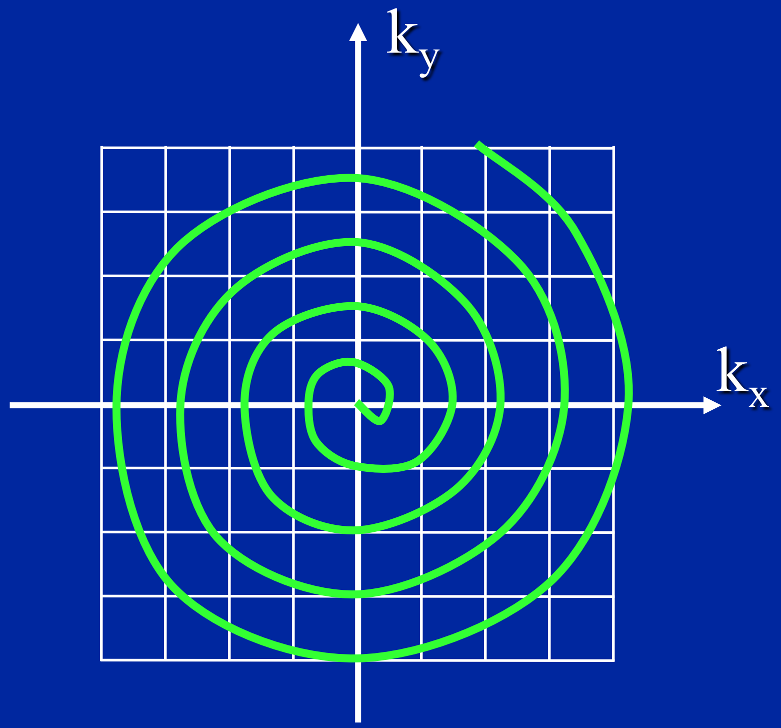


Spiral Pattern



Single-shot spiral, TE = 25 ms,
TR = 2 s, 32 slices

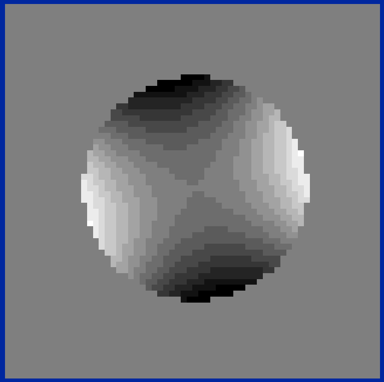
Spiral Imaging

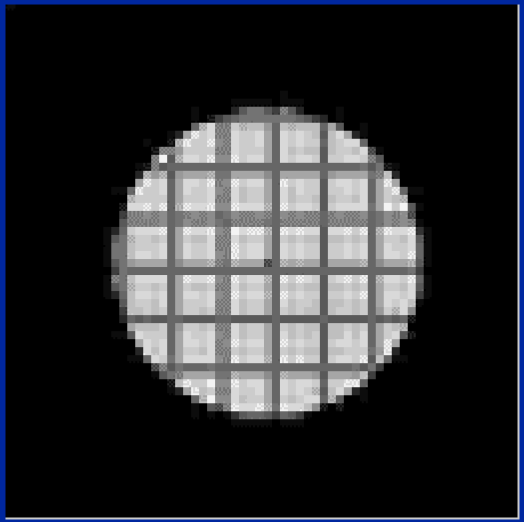


Spiral Pattern

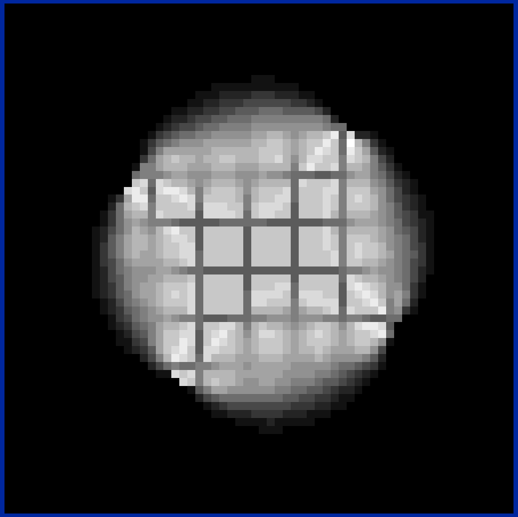
- Single-shot acquisition
- Typically matrix sizes for fMRI
  - 64x64, 96x96
  - 128x128 interleaved
- Acquisition rates
  - TR = 1-2 sec
  - 20-40 slices
- Suffers some artifacts
  - Blurring

Spiral Off-Resonance Distortions





perfect shim



poor shim

Courtesy of P. Jezzard

Single-shot Imaging

- Single-shot imaging is an extremely rapid and useful class of imaging methods.
- It does, however, require high performance hardware. Why?
  - In spin-warp, we acquire a small piece of data for an image with each RF pulse.
  - However in EPI and spiral, we try to acquire all of the data for an image with a single RF pulse.

Single-shot Imaging

- Need powerful gradient amps
- Limitations:
  - Peripheral nerve stimulation
  - Acoustic noise
  - Increased image noise
  - Heating and power consumption in gradient subsystem
- Other issues:
  - Limited spatial resolution
  - Image distortions
  - Some limits on available contrast

Pulse Sequences
(description of image acquisition)

- Two Major Aspects
  - Contrast (Spin Preparation)

> What kind of contrast does the image have?
> What is the TR, TE, Flip Angle, etc.?
> Gradient echo/spin echo/etc.

  - Localization (Image Acquisition)

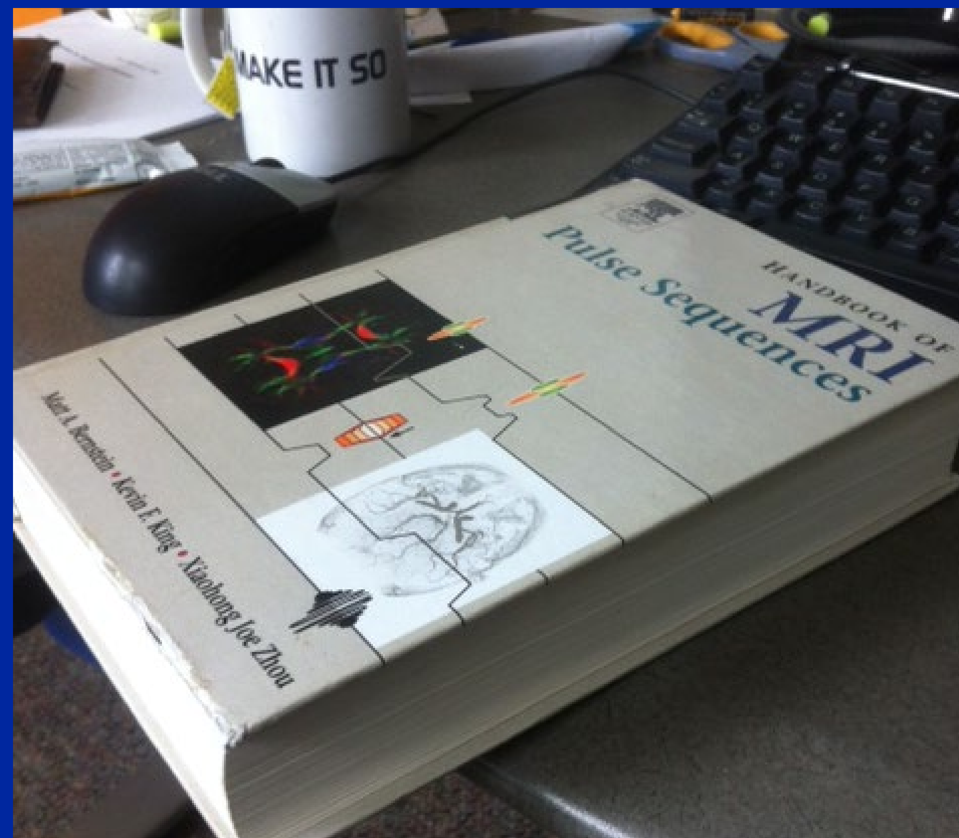
> How is the image acquired?
> How is "k-space" sampled?
> Spatial Resolution?

Pulse Sequences

- Spin Preparation (contrast)
  - Spin Echo (T1, T2, Density)
  - Gradient Echo
  - Inversion Recovery
  - Diffusion
  - Velocity Encoding
- Image Acquisition Method (localization, k-space sampling)
  - Spin-Warp
  - EPI, Spiral
  - RARE, FSE, etc.

Pulse sequences

There are many, many ways to excite spins and sample k-space



Localization vs. Contrast

- In many cases, the localization method and the contrast weighting are independent.
  - For example, the spin-warp method can be used for T1, T2, or nearly any other kind of contrast.
  - T2-weighted images can be acquired with spin-warp, EPI, spiral and RARE pulse sequences.

Localization vs. Contrast

- But, some localization methods are better than others at some kinds of contrast.
  - For example, RARE (FSE) is not very good at generating short-TR, T1-weighted images.
- In general, however, we can think about localization methods and contrast separately.

The 3rd Dimension

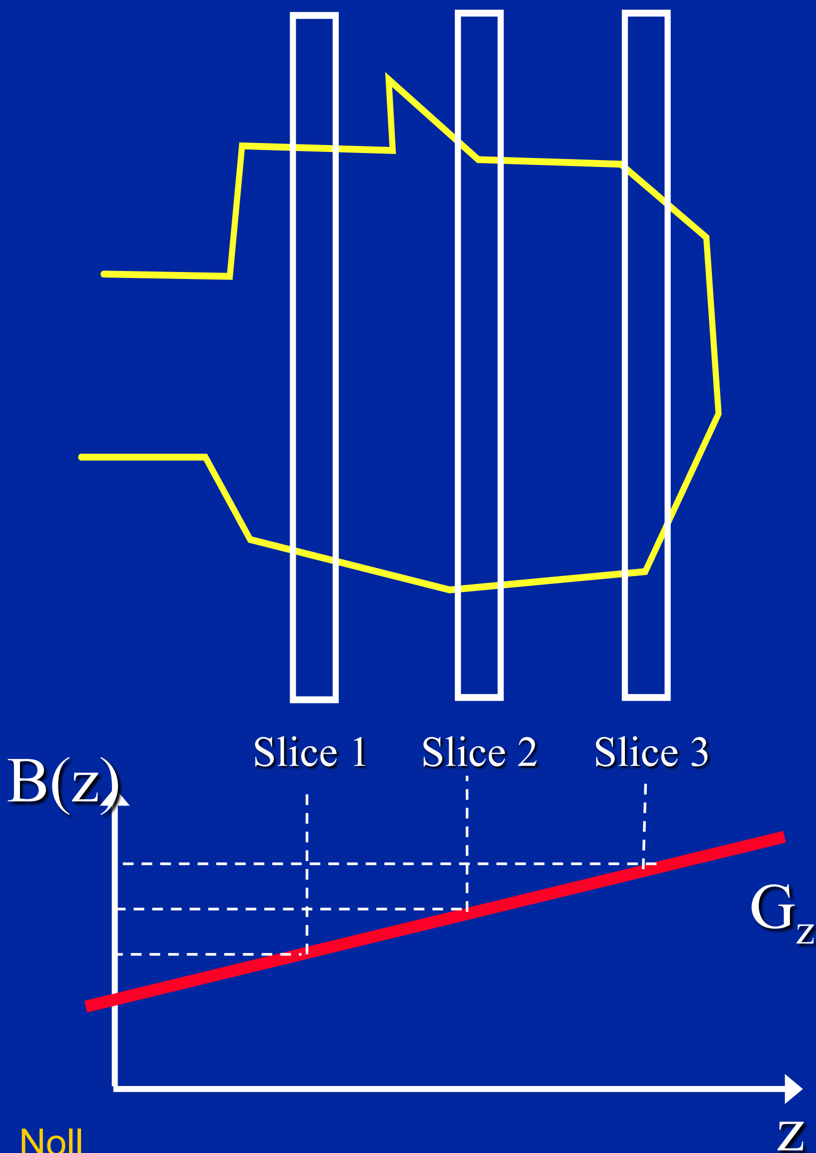
- We've talked about 1D and 2D imaging, but the head is 3D.
- Solution #1 – 3D Imaging
  - Acquire data in a 3D Fourier domain
  - Image is created by using the 3D Fourier transform
  - E.g. 3D spin-warp pulse sequence

- Solution #2 – Slice Selection
  - Excite a 2D plane and do 2D imaging
  - Most common approach

Slice Selection

- The 3rd dimension is localized during excitation
  - "Slice selective excitation"
- Makes use of the resonance phenomenon
  - Only "on-resonant" spins are excited

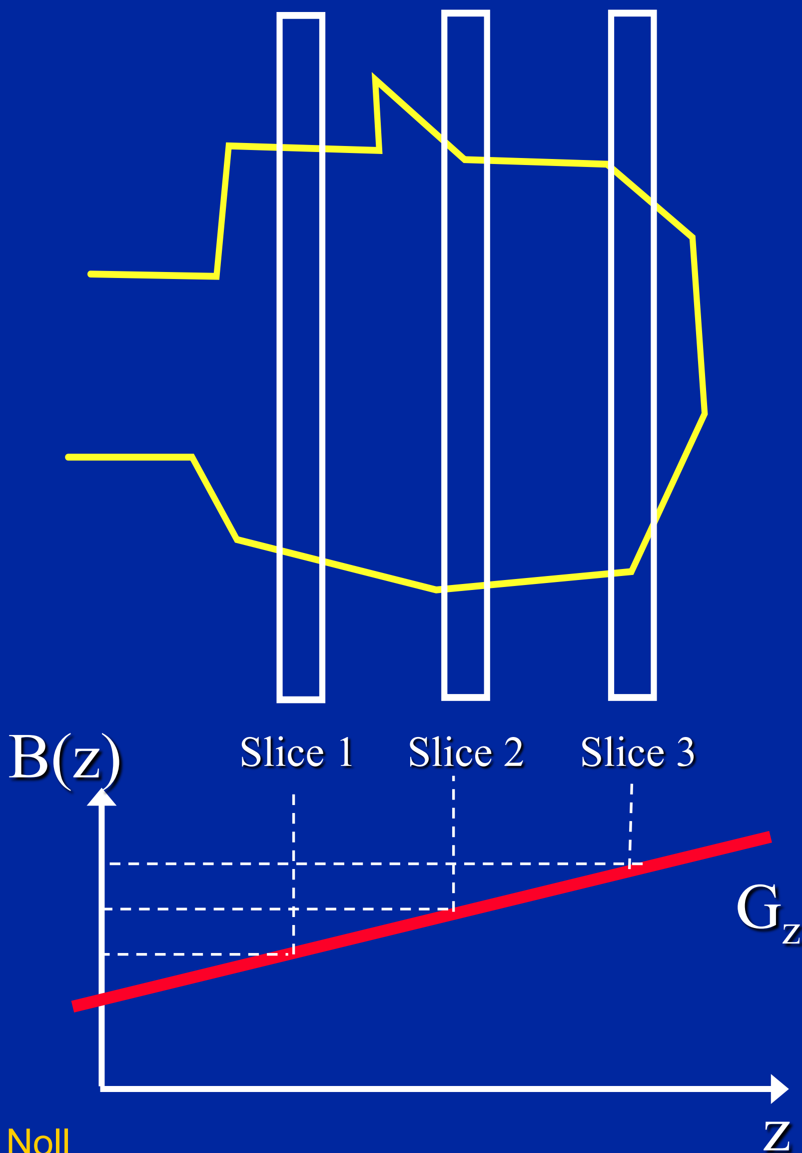
Slice Selection



With the z-gradient on, slices at different z positions have a different magnetic fields and therefore different frequencies :

$$\omega(z_1) < \omega(z_2) < \omega(z_3)$$

Slice Selection



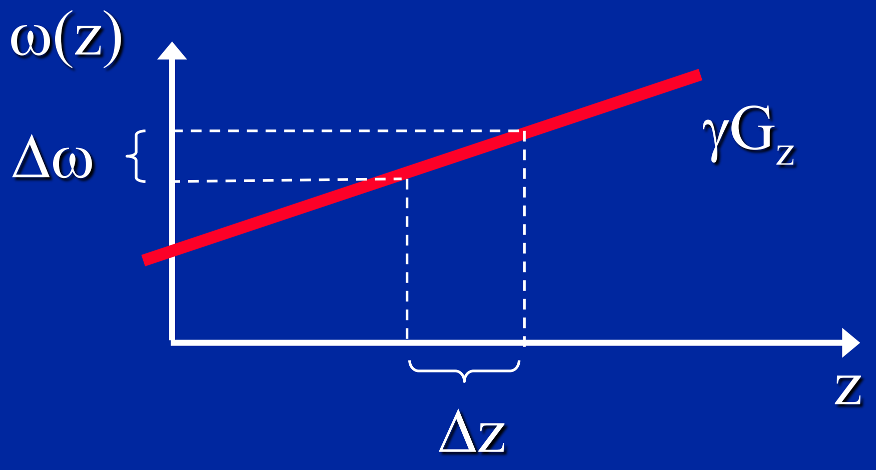
Slice 1 is excited by setting the excitation frequency to $\omega(z_1)$

Slice 2 is excited by setting the excitation frequency to $\omega(z_2)$

Interesting note: Exciting a slice does not perturb relaxation processes that are occurring in the other slices.

Slice Thickness

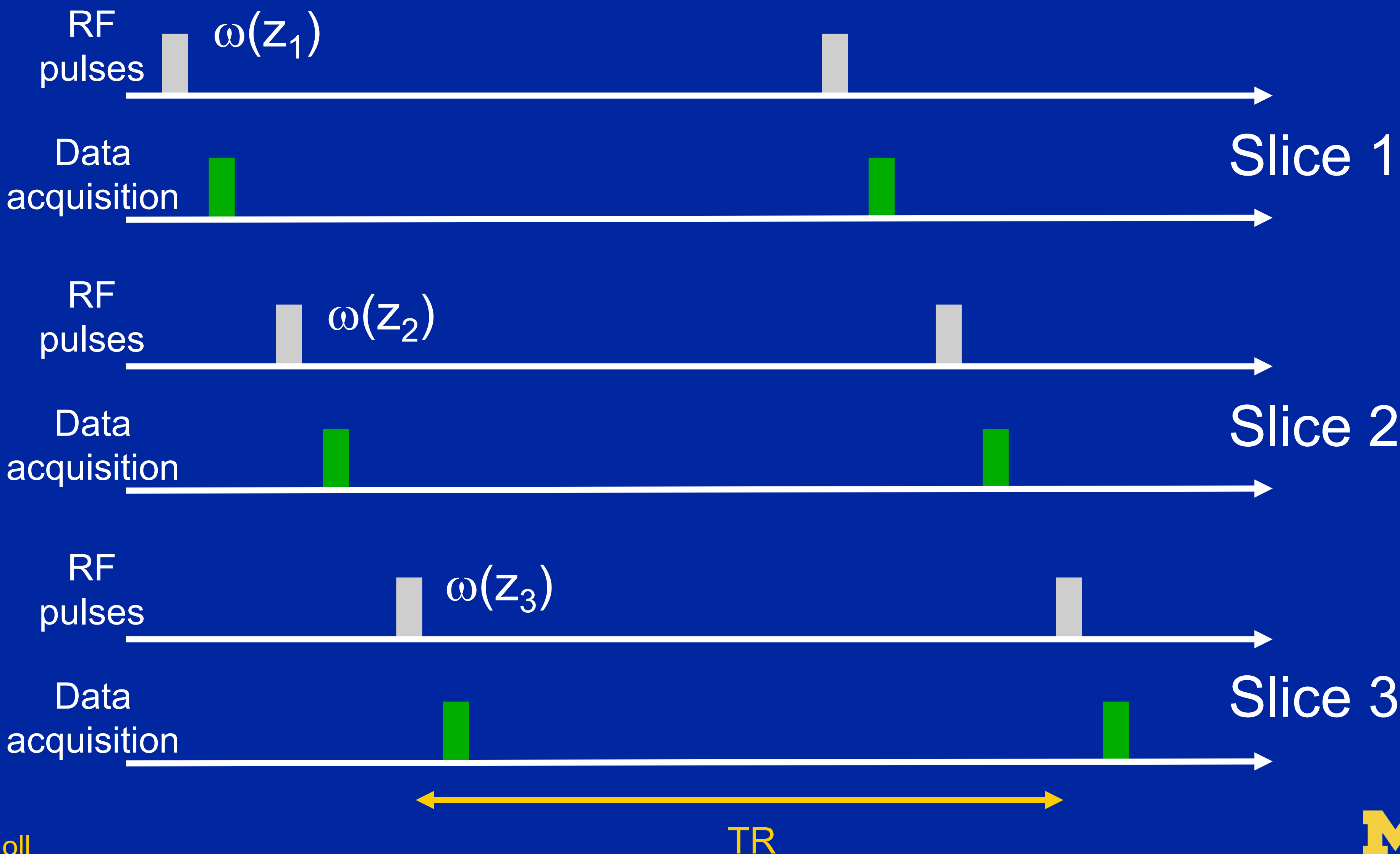
- Slice thickness is adjusted by changing the "bandwidth" of the RF pulse
- Bandwidth ~ 1 / (duration of RF pulse)
  - E.g., for duration = 1 ms, BW ~ 1 kHz



Multi-Slice Imaging

- Since T1's are long, we often would like to have long TR's (500-4000 ms)
- While one slice is recovering (T1), we can image other slices without perturbing the recovery process

Multi-Slice Imaging



Advanced Image Acquisition Topics

- T2 vs. T2*
  - Spin-echo vs. Gradient-echo
- Parallel Imaging (GRAPPA/SENSE/iPAT/ASSET)
- Simultaneous Multi-Slice Imaging

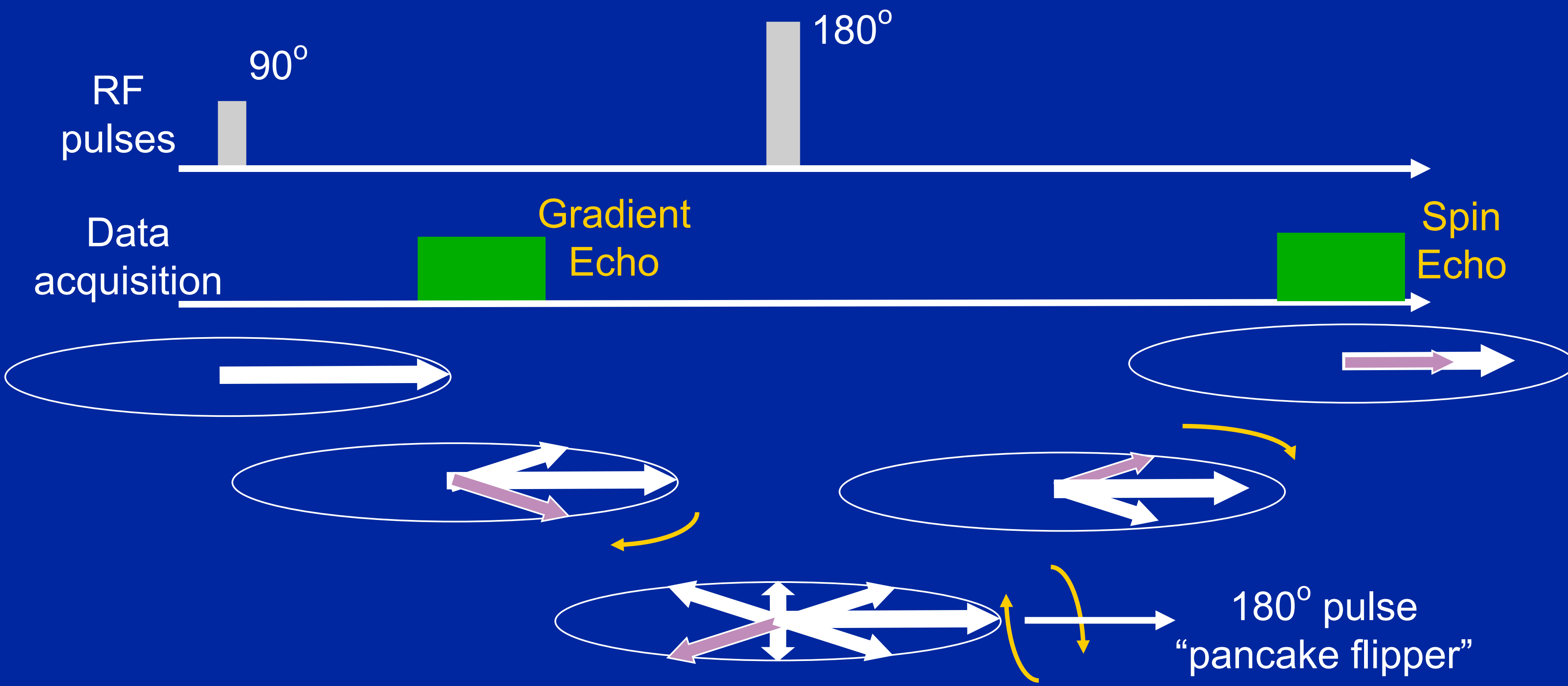
What is T2* (or R2*)?

- T2* has two parts:
  - Inter-molecular interactions leading to dephasing, a.k.a. T2 decay (note: the rate $R2 = 1/T2$)
  - Macroscopic or mesoscopic static magnetic field inhomogeneity leading to dephasing, a.k.a. T2'

$$\frac{1}{T2^*} = \frac{1}{T2'} + \frac{1}{T2} \quad - or - \quad R2^* = R2' + R2$$

- Pulse sequence issues:
  - Spin echoes are sensitive to T2
  - Gradient echoes are sensitive to T2*

Spin-Echo Pulse Sequence

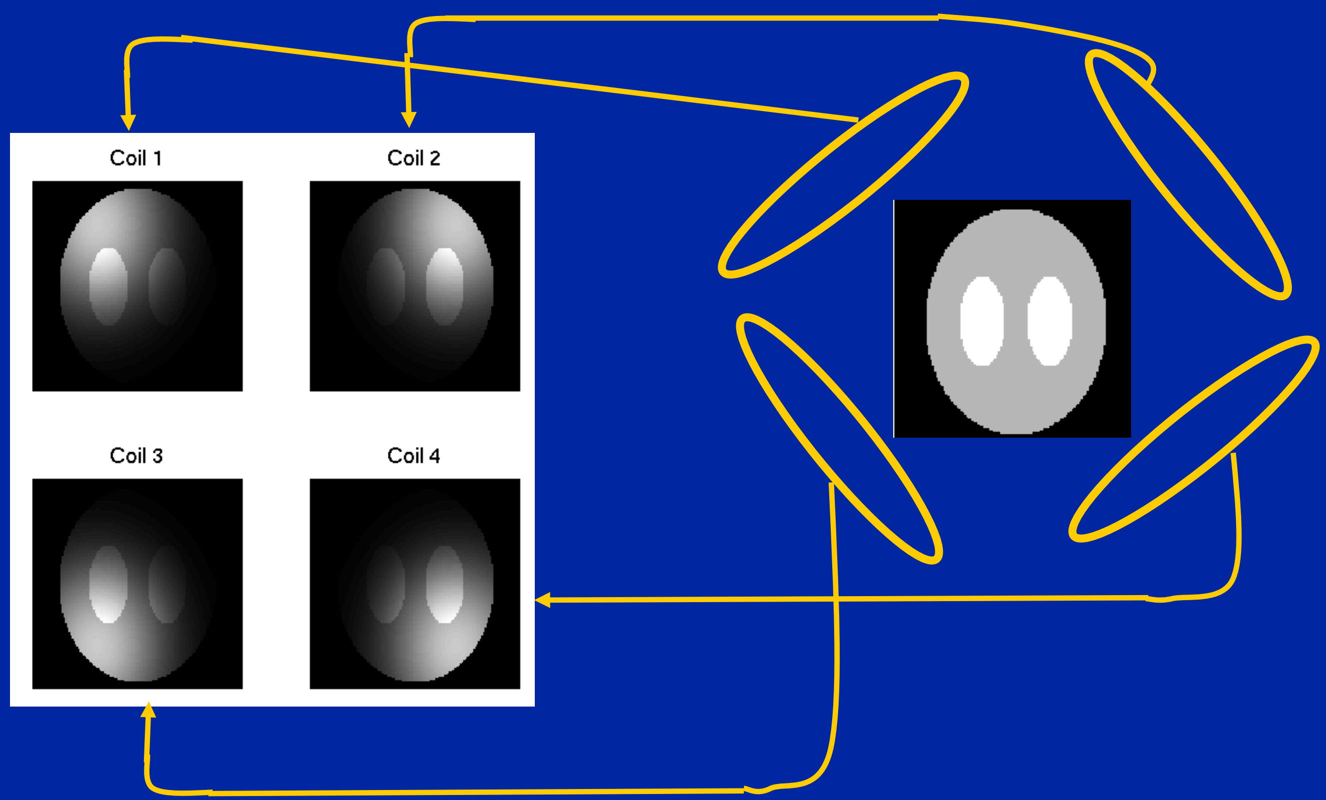


Parallel Imaging

- Basic idea: combining reduced Fourier encoding with coarse coil localization to produce artifact free images
  - Artifacts (aliasing) from reduced Fourier encoding are spatially distinct in manner similar to separation of the coil sensitivity patterns
- Goes by many names. Most common:
- SENSE (SENSitivity Encoding) (also ASSET)
  - Pruessmann, et al. *Magn. Reson. Med.* 1999; **42**: 952-962.
- GRAPPA (GeneRalized Autocalibrating Partially Parallel Acquisitions) (also iPAT, ARC)
  - Griswold, et al. *Magn. Reson. Med.* 2002; **47**: 1202-10.

Localization in MR by Coil Sensitivity

- Coarse localization from parallel receiver channels attached to an array coil

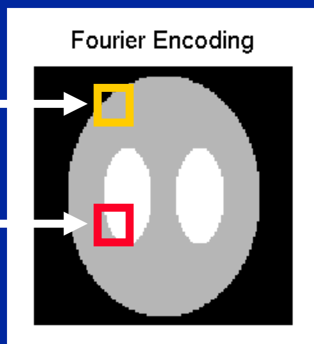


SENSE Imaging – An Example

Full Fourier Encoding
Volume Coil

Pixel A

Pixel B



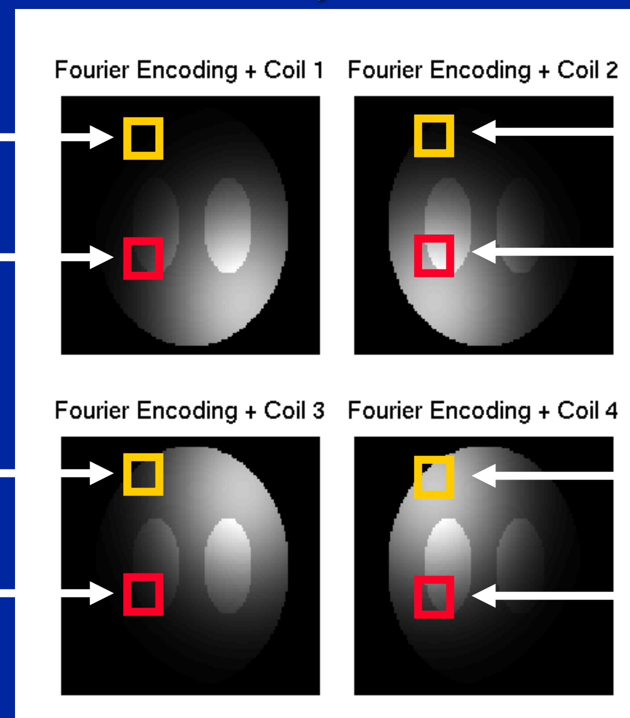
Full Fourier Encoding
Array Coil

$S_{1A}A$

$S_{1B}B$

$S_{3A}A$

$S_{3B}B$



$S_{2A}A$

$S_{2B}B$

$S_{4A}A$

$S_{4B}B$

Unknown Pixel
Values A & B

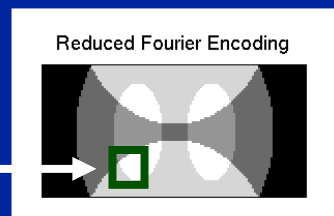
Known Sensitivity
Info S_{1A}, S_{1B},…

SENSE Imaging – An Example

Reduced Fourier – Speed-Up R=2
Volume Coil

Insufficient Data To Determine A & B

A+B

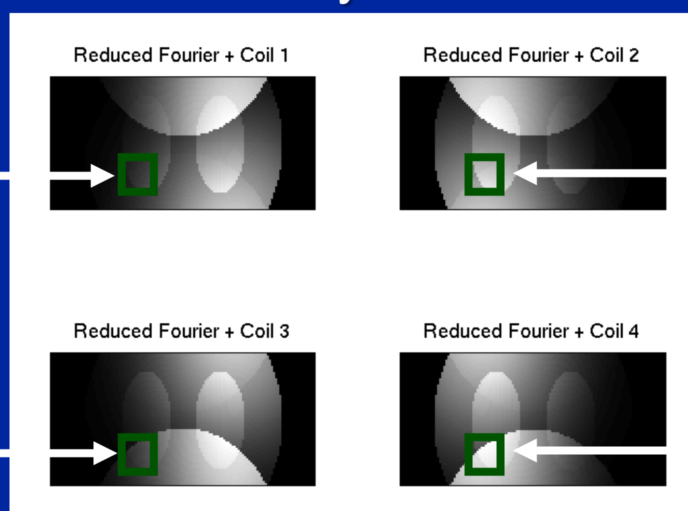


Reduced Fourier – Speed-Up R=2
Array Coil

Extra Coil Measurements Allow Determination of A & B

$S_{1A}A+S_{1B}B$

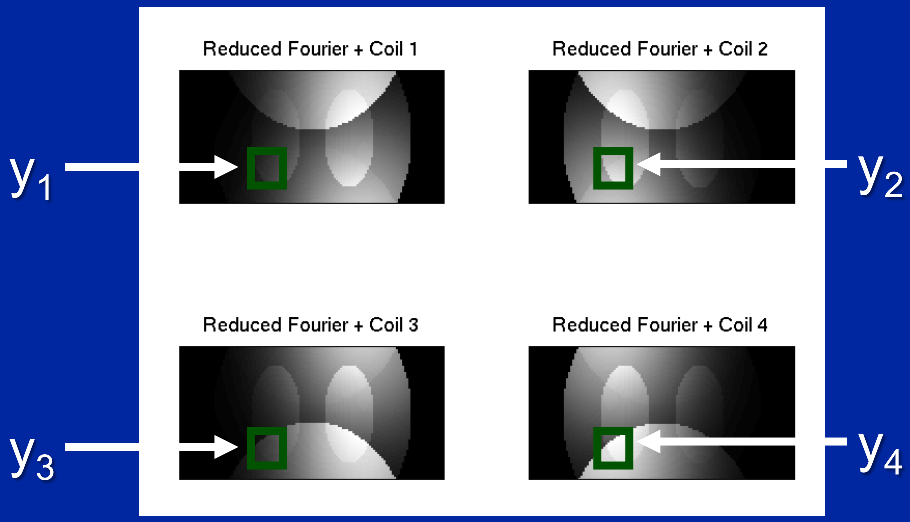
$S_{2A}A+S_{2B}B$



$S_{3A}A+S_{3B}B$

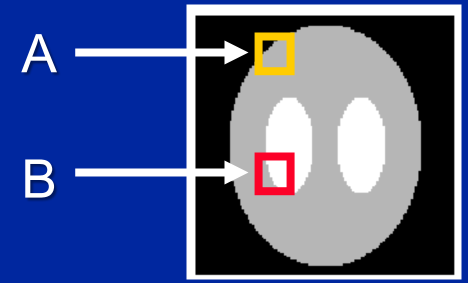
$S_{4A}A+S_{4B}B$

SENSE Imaging – An Example



$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} S_{1A} & S_{1B} \\ S_{2A} & S_{2B} \\ S_{3A} & S_{3B} \\ S_{4A} & S_{4B} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Solving this matrix equation leads to A & B and the unaliased image



Parallel Imaging

- In the lecturer's opinion, parallel imaging is very useful for structural imaging, but only moderately useful for fMRI.
- Pros:
  - Higher spatial resolution
  - Some reduction of distortions
- Cons:
  - But lower SNR
  - Minimal increase in temporal resolution (in fMRI)

Simultaneous Multi-Slice Imaging

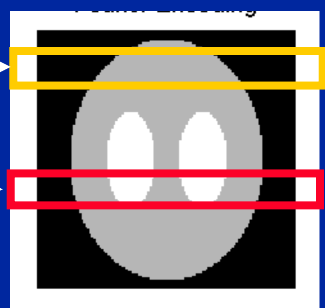
- Basic Idea: Use coil localization information to separate two or more overlapping slices
- Similar to parallel imaging
- References:
  - Larkman, et al. *J. Magn. Reson. Imaging* 2001; **13**: 313-317.
  - Moeller, et al. *Magn. Reson. Med.* 2009; **63**:1144–1153.
  - Setsompop, et al. *Magn. Reson. Med.* 2012; **67**:1210–1224.

Simultaneous Multi-Slice Imaging – An Example

2 Slices
Volume Coil

Slice A

Slice B



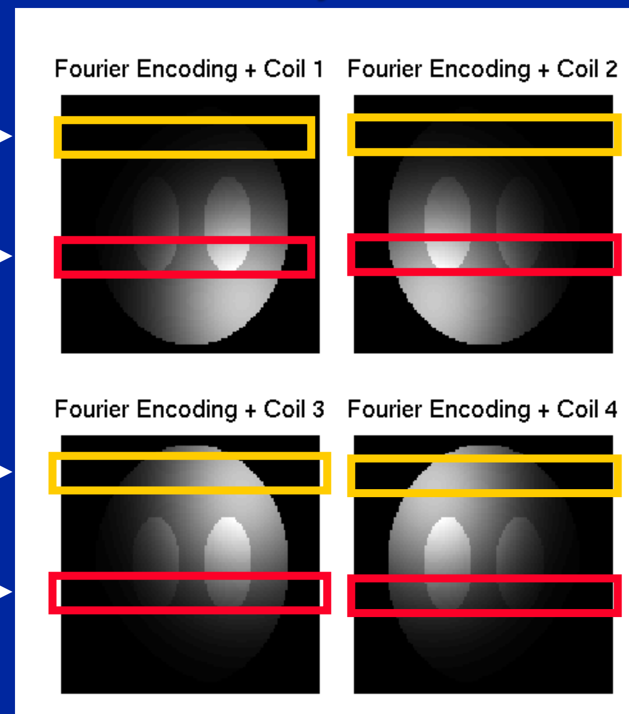
2 Slices
Array Coil

$S_{1A}A$

$S_{1B}B$

$S_{3A}A$

$S_{3B}B$



$S_{2A}A$

$S_{2B}B$

$S_{4A}A$

$S_{4B}B$

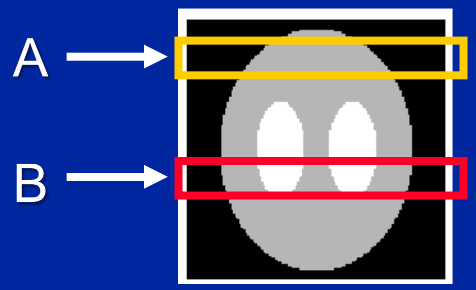
Unknown
Slices A & B

Known Sensitivity
Info S_{1A}, S_{1B},…

Simultaneous Multi-Slice Imaging – An Example

- Same basis equations as parallel imaging
- Operates on slices that overlap instead of aliases of a single slice
- Can be combined with parallel imaging

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} S_{1A} & S_{1B} \\ S_{2A} & S_{2B} \\ S_{3A} & S_{3B} \\ S_{4A} & S_{4B} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$



Simultaneous Multi-Slice Imaging

- Really quite useful for single shot imaging applications like fMRI and diffusion tensor imaging (DTI)
- Pros:
  - Increase in temporal resolution (2x-8x!)
  - Allows for thinner slices
  - Faster acquisition reduced effects of physio noise
- Cons:
  - Small increase in noise, artifact from imperfect decoding of slices